May 1990 Real Analysis — Implications for Neural Networks

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1) Domain D [of functions modeled (approximated) by neural [redacted] network] should be compact.

We cannot use all of $\mathbb{R}^n$ as domain (unbounded, not compact) because none of our convergence theorems will apply.

We need a bounded domain. Formally, we need closure but the distinction between open and closed sets, etc. is usually not relevant in practical situations. (It's too hard to tell if the domain is open or closed or etc. in practice.) We can safely assume [redacted] a closed domain.

2) If a network can approximate continuous functions (with uniformly convergent sequence of functions), then the network can approximate measurable functions (with uniformly convergent sequence of functions), over all but a vanishingly small portion of the domain.

Thus, networks satisfying Stone-Weierstrass thm can approximate measurable funcs. Uniform convergence.

Since stepped functions (i.e. Σ steps) can approximate measurable functions with the same kind of convergence results as for continuous funcs, any network capable of approximating stepped functions (namely 2-layer sigmoid-linear net) can approximate measurable functions.

3) Virtually every function imaginable is measurable. Discontinuous functions are measurable (usually). A step function is discontinuous but measurable. Delta functions may be thought of as the limit of sequences of continuous functions and may be modeled acceptably by continuous functions. The delta function is measurable.