~~tool~~: Linear regression is a least squares method where the model function is a straight line.

$$f(\vec{a}, x) = a_1 x + a_2$$

We find the least squares equations as follows:

$$\left.\frac{d f(\vec{a}, x)}{d a_1}\right|_{x=x_i} = x_i$$

$$\left.\frac{d f(\vec{a}, x)}{d a_2}\right|_{x=x_i} = 1$$

For data points $(x_1, y_1), \ldots, (x_N, y_N)$ we solve for a_1 and a_2:

$$\sum_{i=1}^{N} (a_1 x_i + a_2 - y_i) x_i = 0$$

and $$\sum_{i=1}^{N} (a_1 x_i + a_2 - y_i) = 0$$

Rearrange: $$a_1 \sum_{i=1}^{N} x_i^2 + a_2 \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} y_i x_i = 0$$

$$a_1 \sum_{i=1}^{N} x_i + a_2 \sum_{i=1}^{N} 1 - \sum_{i=1}^{N} y_i = 0$$

Write as matrix eq'n:

$$\begin{bmatrix} \sum_{i=1}^{N} x_i^2 & \sum_{i=1}^{N} x_i \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} y_i x_i \\ \sum_{i=1}^{N} y_i \end{bmatrix}$$

or $$X_\Sigma \quad \vec{a} = \vec{y}_\Sigma$$

$$\vec{a} = X_\Sigma^{-1} \vec{y}_\Sigma$$

where $$X_\Sigma^{-1} = \frac{1}{\sum_{i=1}^{N} x_i^2 \sum_{i=1}^{N} 1 - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i} \begin{bmatrix} \sum_{i=1}^{N} 1 & -\sum_{i=1}^{N} x_i \\ -\sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \end{bmatrix}$$