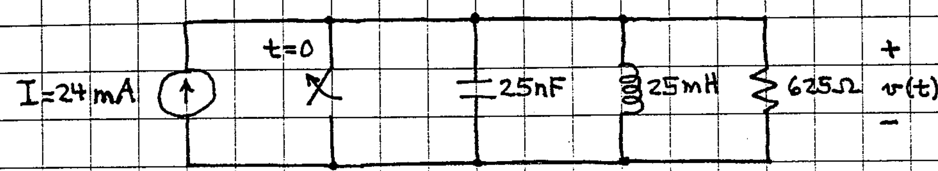

Initial stored energy = 0 J

Find $i_L(t \geq 0)$.

ans: $i_L(t \geq 0) = 24 - 24e^{-32kt} \cos 24kt - 32e^{-32kt} \sin 24kt$ mA

sol'n: We can use superposition to find $i_L(t \geq 0)$ as follows:

1) Find a constant solution $i_{L1}(t \geq 0) = I$ that is equal to the driving term on the righthand side of the differential eq'n:

$$i_L + \frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = I$$

Note that $\frac{di_L}{dt} = 0$ and $\frac{d^2 i_L}{dt^2} = 0$, so we have $i_L(\geq 0) = I$ is a sol'n of the diffential eq'n.

2) Find a general solution $i_{L2}(t \geq 0)$ such that $i_{L2}(t=0) = \underbrace{i_L(t=0)}_{\text{init cond}} - \underbrace{i_{L1}(t=0)}_{\text{prev sol'n}}$

or " $= \underbrace{i_L(t=0)}_{\text{init cond}} - I$

and such that $\left.\frac{d\, i_{L2}(t)}{dt}\right|_{t=0} = 0$;

(and we note that $i_{L2}(t \geq 0)$ is a general solution that solves the differential eq'n $i_L + \frac{L}{R}\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} = 0$)

3) Find a general solution $i_{L3}(t \geq 0)$ such that $i_{L3}(t=0) = 0$

<u>and</u> such that $\left.\frac{d\,i_{L3}(t)}{dt}\right|_{t=0} = \underbrace{\left.\frac{d\,i_L(t)}{dt}\right|_{t=0}}_{\text{init cond}}$

Summary: $i_{L1}(t \geq 0)$ solves for driving term I

$i_{L2}(t \geq 0)$ solves for initial $i_L(t=0)$ but contributes nothing to $\left.\frac{di_L}{dt}\right|_{t=0}$.

$i_{L3}(t \geq 0)$ solves for initial $\left.\frac{di_L}{dt}\right|_{t=0}$ but contributes nothing to $i_L(t=0)$.

By superposition, the complete solution for $i_L(t \geq 0)$ is

$$i_L(t \geq 0) = i_{L1}(t \geq 0) + i_{L2}(t \geq 0) + i_{L3}(t \geq 0)$$

Q. What is the advantage?

A. We can now find symbolic sol'ns for $i_{L2}(t \geq 0)$ and $i_{L3}(t \geq 0)$ that avoid the problem of solving two eq'ns in two unknowns.

We already have $i_{L1}(t \geq 0) = I$.

(Now Assume <u>not</u> critically damped.)

For $i_{L2}(t \geq 0)$ we have $\left. i_{L2}(t \geq 0)\right|_{t=0} = \left. A_1 e^{s_1 t} + A_2 e^{s_2 t}\right|_{t=0} = i_L(t=0) - I$.

$$\therefore \quad A_1 e^{s_1 \cdot 0} + A_2 e^{s_2 \cdot 0} = i_L(t=0) - I$$

$$\text{or} \quad A_1 + A_2 = i_L(t=0) - I$$

<u>and</u> $\left.\frac{d}{dt} i_{L2}(t \geq 0)\right|_{t=0} = \left. A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}\right|_{t=0} = 0$

$$\text{or} \quad A_1 s_1 + A_2 s_2 = 0$$

From the preceding eq'n we have $A_2 = -A_1 \frac{s_1}{s_2}$.

The first eq'n becomes $A_1 - A_1 \frac{s_1}{s_2} = i_L(t=0) - I$

or $A_1 \frac{s_2 - s_1}{s_2} = i_L(t=0) - I$

or $\boxed{A_1 = \frac{s_2}{s_2 - s_1}\left[i_L(t=0) - I\right]}$

For $i_{L3}(t \geq 0)$ we have $i_{L3}(t \geq 0)\Big|_{t=0} = E_1 e^{s_1 t} + E_2 e^{s_2 t}\Big|_{t=0} = 0$

or $E_1 + E_2 = 0$

and $\frac{d i_{L3}(t \geq 0)}{dt}\Big|_{t=0} = E_1 s_1 e^{s_1 t} + E_2 s_2 e^{s_2 t}\Big|_{t=0} = \underbrace{\frac{d i_L(t)}{dt}\Big|_{t=0}}_{\text{init cond}}$

or $E_1 s_1 + E_2 s_2 = \frac{d i_L(t)}{dt}\Big|_{t=0}$

From the first eq'n we have $E_2 = -E_1$.

The second eq'n becomes $E_1(s_1 - s_2) = \frac{d i_L(t)}{dt}\Big|_{t=0}$

or $\boxed{E_1 = \frac{1}{s_1 - s_2} \frac{d i_L(t)}{dt}\Big|_{t=0}}$

Now we can write down $i_{L2}(t \geq 0)$ and $i_{L3}(t \geq 0)$:

$i_{L2}(t \geq 0) = A_1 e^{s_1 t} - A_1 \frac{s_1}{s_2} e^{s_2 t}$ A_1 given by boxed eq'n above.

$i_{L3}(t \geq 0) = E_1 e^{s_1 t} - E_1 e^{s_2 t}$ E_1 given by boxed eq'n above.

Our complete soln is:

$$i_L(t) = i_{L1}(t \geq 0) + i_{L2}(t \geq 0) + i_{L3}(t \geq 0)$$

$$\boxed{i_L(t) = I + (A_1 + E_1)e^{s_1 t} - \left(A_1 \frac{s_1}{s_2} + E_1\right)e^{s_2 t}}$$

Now we apply the formulas to Example 8.7.

$I = 24\,mA$ $\quad i_L(t=0) = 0A$ since no energy stored in circuit

To find $\left.\frac{di_L(t)}{dt}\right|_{t=0}$ we use $v_L(t=0) = L\left.\frac{di_L(t)}{dt}\right|_{t=0}$

But $v_L(t=0) = v_C(t=0) = 0V$ since no energy stored in circuit

From these values, we conclude that:

$$A_1 = \frac{s_2}{s_2 - s_1}(-I) = \frac{s_2}{s_2 - s_1}(-24mA)$$

$$E_1 = \frac{1}{s_1 - s_2} \cdot 0 = 0$$

Now for characteristic roots s_1 and s_2:

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 625\Omega \cdot 25nF} = 32k \text{ rad/s}$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{25mH \cdot 25nF} = (40k \text{ rad/s})^2$$

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -32k \pm j24k \text{ rad/s}$$

$$s_2 - s_1 = -2\sqrt{\alpha^2 - \omega_0^2} = -j48k \text{ rad/s}$$

Write s_1, s_2 in polar form to facilitate division:

$$s_1 = 40k\, e^{j143^\circ} \qquad s_2 = s_1^* = 40k\, e^{-j143^\circ}$$

$$\frac{s_2}{s_2 - s_1} = \frac{40k\, e^{-j143^\circ}}{-j48k} = j\frac{40k\, e^{-j143^\circ}}{48k} = \frac{e^{j90^\circ}\, 40k\, e^{-j143^\circ}}{48k}$$

$$'' = \frac{5}{6} e^{-j53^\circ}$$

$$\frac{s_1}{s_2} = \frac{40k\, e^{j143^\circ}}{40k\, e^{-j143^\circ}} = e^{j286^\circ} = e^{-j74^\circ}$$

$$A_1 = \frac{5}{6} e^{-j53^\circ}(-24\text{mA}) = -20\, e^{-j53^\circ} \text{ mA}$$

$$'' = 20\, e^{j180^\circ} e^{-j53^\circ} \text{ mA} = 20\, e^{j127^\circ} \text{ mA}$$

$$A_1 \frac{s_1}{s_2} = 20 e^{j127^\circ} e^{-j74^\circ} \text{ mA} = 20\, e^{j53^\circ} \text{ mA}$$

In rectangular form $A_1 = -12 + j16$ mA
$A_2 = 12 + j16$ mA

Using $e^{s_1 t} = e^{-\alpha t} e^{j\omega_d t}$ $\quad e^{s_2 t} = e^{-\alpha t} e^{-j\omega_d t}$ gives:

$$i_L(t \geq 0) = 24\text{mA} + (-12 + j16)\text{mA}\; e^{-32kt}\big(e^{j24kt} = \cos 24kt + j \sin 24kt\big)$$

$$- (12 + j16)\text{mA}\; e^{-32kt}\big(e^{-j24kt} = \cos 24kt - j \sin 24kt\big)$$

$$= 24\text{mA} + e^{-32kt}(-2 \cdot 12 \cdot \cos 24kt - 2 \cdot 16 \cdot \sin 24kt)\text{ mA}$$

$$\therefore\; i_L(t \geq 0) = 24 - 24 e^{-32kt} \cos 24kt - 32 e^{-32kt} \sin 24kt \text{ mA}$$

Comment: The over-damped and critically damped cases are cleaner than the above because they involve only real #'s.

Note: We avoided 2 eq'ns in 2 unknowns, as advertized.