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Ref: Nilsson & Riedel, Rev 6, Prob 9.52

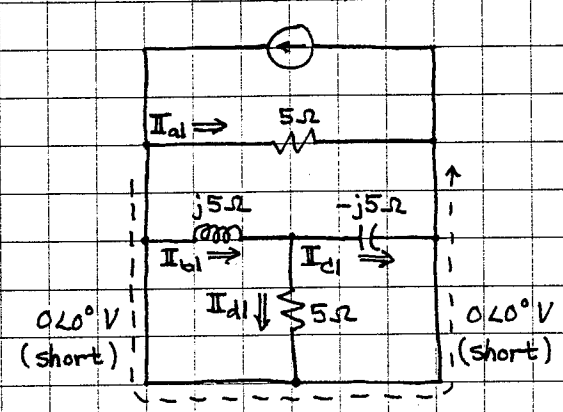
SUPERPOSITION
CIRCUITS
$V_{AC} + V_{AC}$
EXAMPLE

ex: (See circuit diagram for mesh-current soln.)

Use superposition to find $\mathbb{I}_a$, $\mathbb{I}_b$, $\mathbb{I}_c$, and $\mathbb{I}_d$.

soln: Turn on one independent source at a time.
Sum currents or voltages from each such circuit.

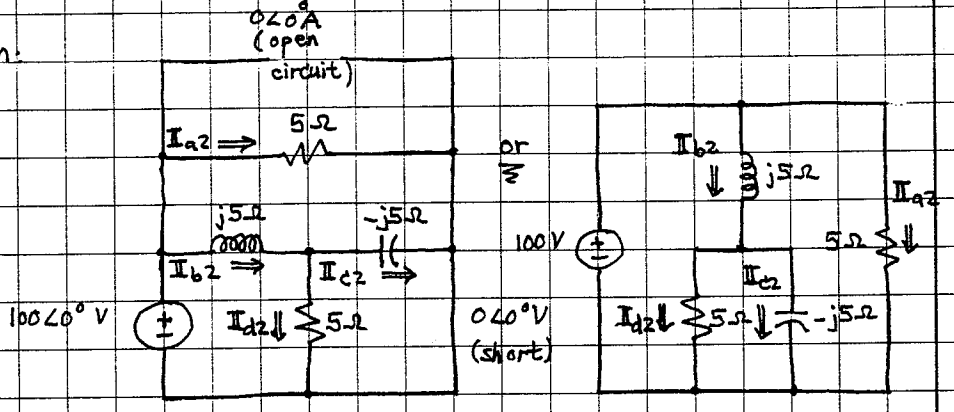
$2\angle 0°$ A on:



Observe that all the current will flow thru the path indicated by the dashed line.

$\therefore \mathbb{I}_{a1} = 0\,A \quad \mathbb{I}_{b1} = 0\,A \quad \mathbb{I}_{c1} = 0A \quad \mathbb{I}_{d1} = 0\,A$

$100\angle 0°$ V on:



$$\mathbb{I}_{a2} = \frac{100\,V}{5\,\Omega} = 20\,A \quad \text{(since } 5\,\Omega \text{ across } 100V \text{ source)}$$

$$\mathbb{I}_{b2} = \frac{100V}{j5\,\Omega + 5\,\Omega \| (-j5)\Omega} \quad \text{(since 3 components across } 100V\text{)}$$

$$5\,\Omega \| -j5\,\Omega = 5\,\Omega \cdot 1 \| -j = 5\,\Omega \frac{-j}{1-j}$$

$$\frac{-j}{1-j} = \frac{-j(1+j)}{(1-j)(1+j)} = \frac{1-j}{2}$$

$$= \frac{20}{j + \frac{1-j}{2}}\,A$$

$$\mathbb{I}_{b2} = \frac{40}{j2 + 1 - j}\ A = \frac{40}{1+j} = \frac{40\,(1-j)}{(1+j)(1-j)} = \frac{40}{2}(1-j) = 20 - j20\ A$$

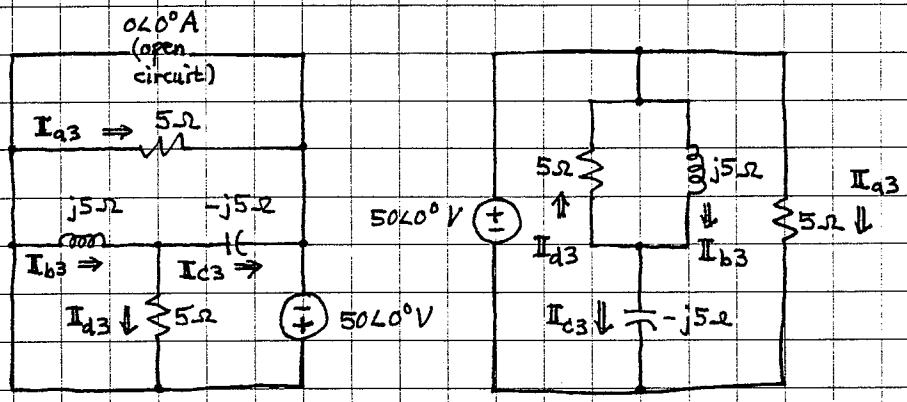
$$\mathbb{I}_{c2} = \mathbb{I}_{b2}\ \frac{5\Omega}{5 - j5\Omega} \qquad (\mathbb{I}\text{-divider})$$

$$= \frac{20\,(1-j)\ \cancel{5}}{(1-j)\ \cancel{5}} = 20\ A$$

$$\mathbb{I}_{d2} = \mathbb{I}_{b2}\ \frac{-j5\Omega}{5 - j5\Omega} \qquad (\mathbb{I}\text{-divider})$$

$$= \frac{20\,(1-j)}{(1-j)}\ \frac{-j\ \cancel{5}}{\cancel{5}} = -j20\ A$$

$50\angle 0°\ V$ on:



$$\mathbb{I}_{a3} = \frac{50\angle 0°\ V}{5\Omega} = 10\ A$$

$$\mathbb{I}_{c3} = \frac{50\angle 0°\ V}{5\Omega \,\|\, j5\Omega + -j5\Omega} = \frac{10}{1 \,\|\, j - j}\ A = \frac{10}{\frac{j}{1+j} - j} = \frac{10\,(1+j)\ A}{j - j(1+j)}$$

$$= 10 + j10\ A$$

$$\mathbb{I}_{b3} = \mathbb{I}_{c3}\ \frac{5\Omega}{5\Omega + j5\Omega} = 10\ \frac{(1+j)}{(1+j)}\ \frac{\cancel{5}}{\cancel{5}} = 10\ A$$

$$\mathbb{I}_{d3} = -\mathbb{I}_{c3}\frac{j5\Omega}{5\Omega + j5\Omega} = -\frac{10\,(1+j)\,\frac{5}{5}\,j}{(1+j)} = -j10\,A$$

Now sum the currents from each circuit to get total $\mathbb{I}$'s:

$$\mathbb{I}_a = \mathbb{I}_{a1} + \mathbb{I}_{a2} + \mathbb{I}_{a3} = 0A + 20A + 10A = 30\,A$$

$$\mathbb{I}_b = \mathbb{I}_{b1} + \mathbb{I}_{b2} + \mathbb{I}_{b3} = 0A + 20 - j20\,A + 10A = 30 - j20\,A$$

$$\mathbb{I}_c = \mathbb{I}_{c1} + \mathbb{I}_{c2} + \mathbb{I}_{c3} = 0A + 20A + 10 + j10A = 30 + j10A$$

$$\mathbb{I}_d = \mathbb{I}_{d1} + \mathbb{I}_{d2} + \mathbb{I}_{d3} = 0A + -j20A + -j10A = -j30\,A$$

Our answers agree with the mesh-current method.

Note how the superposition method reveals that the $2\angle 0°\,A$ source adds nothing to currents $\mathbb{I}_a$, $\mathbb{I}_b$, $\mathbb{I}_c$, and $\mathbb{I}_d$.