By: Neil E. Cotter

Ref: Nilsson & Riedel, Rev 6, Prob 9.62

Given load: 150 Ω and 20 nF in series

linear transformer: $R_1 = 12\,\Omega$ $L_1 = 80\,\mu H$

$R_2 = 50\,\Omega$ $L_2 = 500\,\mu H$

$M = 100\,\mu H$

sinusoidal V-source: $\omega = 500$ krad/s

$Z_{internal}$ @ $\omega = 500$ krad/s $= 5 + j16\,\Omega$

$V_{no\ load} = 125\ V$

Find a) value of impedance reflected into primary

b) value " " seen from terminals of V source

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ans: a) $8 - j6\,\Omega$

b) $20 + j34\,\Omega$

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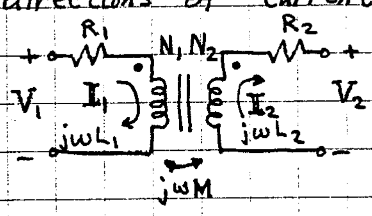
sol'n: a) We use the linear transformer model, and we observe that $V_{no\ load}$ = Voltage of source in model.

Note: We use standard position of dots and directions of currents for our transformer:

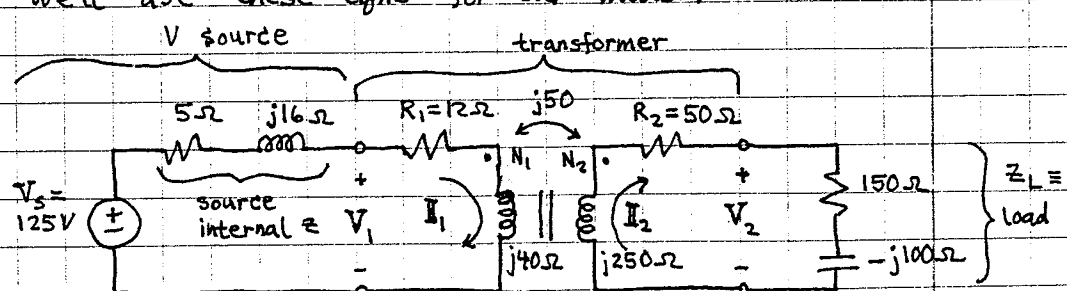


The equations for the above transformer are

$$V_1 = (R_1 + j\omega L_1)\mathbb{I}_1 - j\omega M \mathbb{I}_2$$
$$V_2 = j\omega M \mathbb{I}_1 - (R_2 + j\omega L_2)\mathbb{I}_2$$

We'll use these eqns for our model.

model:



(see next page for component value calcs.)

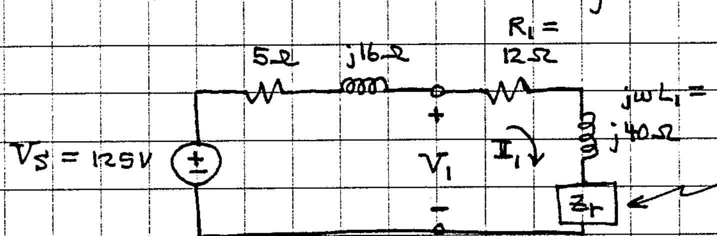
$$\frac{1}{j\omega C} = -j\frac{1}{500k \cdot 20n} = -j100$$

$$j\omega L_1 = j\,500k \cdot 80\mu = j\,40k\,k\,\mu = j40$$

$$j\omega L_2 = j\,500k\;500\mu = j\,250\,kk\mu = j250$$

$$j\omega M = j\,500k\;100\mu = j\,50\,kk\mu = j50$$

Reflected impedance Z_r is the impedance we would place in series with the primary side of the transformer to get the same V_1 versus I_1 characteristics as the original circuit:



reflected Z (instead of including secondary side)

The Text calculates $Z_r = \dfrac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$.

We'll derive this result, too:

For the secondary we have sum of V drops around loop:

$$V_2 = Z_L I_2 = \underbrace{j\omega M I_1 - (R_2 + j\omega L_2) I_2}_{\text{from transformer eq'ns}}$$

or $$(R_2 + j\omega L_2 + Z_L) I_2 = j\omega M I_1$$

or $$I_2 = \frac{j\omega M}{R_2 + j\omega L_2 + Z_L} I_1$$

From the first transformer eq'n, with this result substituted for I_2, we have:

$$V_1 = (R_1 + j\omega L_1) I_1 - j\omega M \overbrace{\frac{j\omega M}{R_2 + j\omega L_2 + Z_L}}^{I_2} I_1$$

or $$\mathbb{V}_1 = (R_1 + j\omega L_1)\mathbb{I}_1 + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + z_L}\mathbb{I}_1$$

For the reflected impedance model we have

$$\mathbb{V}_1 = (R_1 + j\omega L_1)\mathbb{I}_1 + z_r\,\mathbb{I}_1$$

Equating expressions for $\mathbb{V}_1$ gives:

$$z_r = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + z_L}$$ ✓

Now we plug in values to obtain the answer to (a):

$$z_r = \frac{(500\text{k})^2 (100\mu)^2}{50 + j250 + 150 - j100} = \frac{250\text{K}\cancel{\text{K}^2}\cdot 10\text{K}\cancel{\mu^2}}{200 + j150}$$

$$= \frac{2.5\text{ K}}{50(4+j3)} = \frac{50}{4+j3} = \frac{50\,(4-j3)}{4^2+3^2}$$

$$= \frac{50}{25}(4-j3) = 2(4-j3)$$

$$z_r = 8 - j6\ \Omega$$

b) The impedance seen from the terminals of the V source is the impedance seen to the right of the terminals where we measure $\mathbb{V}_1$.

In other words, we are finding the impedance seen looking into the transformer.

From the reflected impedance model we see that this is

$$z = R_1 + j\omega L_1 + z_r = 12 + j40 + 8 - j6 = 20 + j34\ \Omega$$