EE 5961-2 / 6961-2 Lecture # 24 Wavelets

6 Mar 2000
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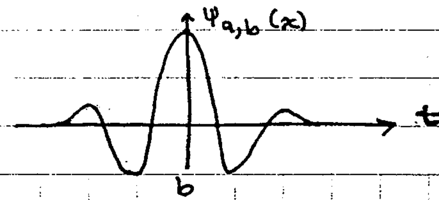
Wavelets

def: wavelet $\equiv$ a function we may scale and time shift to form an orthonormal basis that is complete.

$$\equiv \quad \psi_{a,b}(t) = \frac{1}{\sqrt{a}}\,\psi\!\left(\frac{t-b}{a}\right)$$



$b \equiv$ time shift
$a \equiv$ time scaling factor

Note: $\frac{1}{\sqrt{a}}$ factor preserves norm of one:

$$\int_{-\infty}^{\infty} \psi_{a,b}^2(t)\, dt \equiv \langle \psi_{a,b}(t), \psi_{a,b}(t) \rangle = 1$$

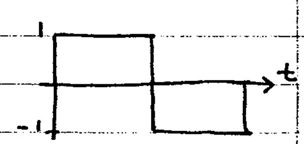
tool: Since $\{\psi_{a,b}(t)\}$ is a complete orthonormal basis we can expand $f(t)$ in terms these wavelets:

$$f(t) = \sum_a \sum_{b(a)} w_{a,b}\, \psi_{a,b}(t)$$

Note: The time shifts, $b(a)$, are a function of what scaling factor, 'a', we are at. We use multiple "a's" or resolutions (in fact infinitely many) from DC to very high (∞ in the limit).

ex: Haar wavelets

$$\psi(t) = \begin{cases} 1 & 0 \le t < \frac{1}{2} \\ -1 & \frac{1}{2} \le t < 1 \\ 0 & \text{otherwise} \end{cases}$$



Use basis $\psi_{m,n}(t) = 2^{-m/2}\,\psi(2^{-m}t - n) \quad m,n \in \mathbb{Z}$

We use all integer m,n pairs, including $m,n < 0$.

ref: Wavelets and Subband Coding, Martin Vetterli and Jelena Kovačević, Prentice Hall, Upper Saddle River: NJ, 1995.
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tool: Since $\{\psi_{a,b}(t)\}$ is a complete orthonormal basis, we find coefficients for wavelet expansion by using inner products.

$$f(t) = \sum_{a} \sum_{b(a)} w_{a,b}\, \psi_{a,b}(t)$$

$$w_{a,b} = \langle f(t), \psi_{a,b}(t) \rangle$$