CIRCUIT: The circuit shown below in Fig. 1 produces a rectangular wave with frequency controlled by potentiometer, R_a, R_b. The duty cycle varies with v_{ctrl}. For an op-amp such as the LM324 with asymmetric rail voltages, the duty cycle will be greater than 50% when the control voltage is at reference. A control voltage halfway between the rail voltages, however, does produce a 50% duty cycle waveform.

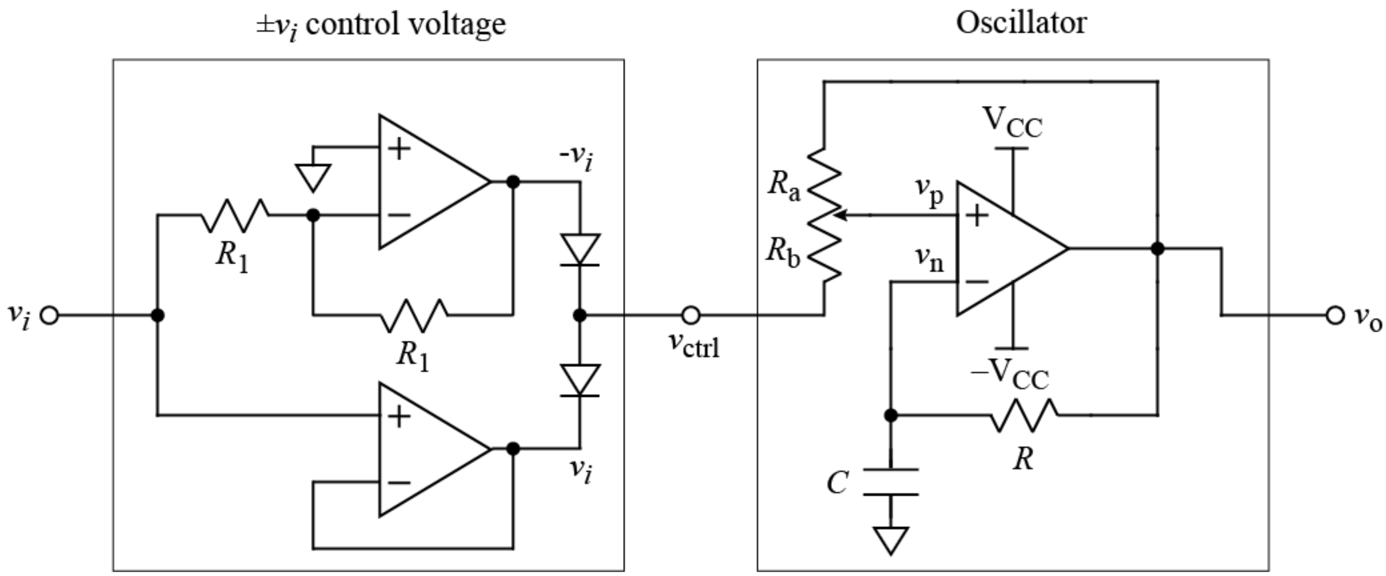

Fig. 1. Voltage-controlled Oscillator.

The first part of the circuit produces a voltage reference, v_{ctrl}, that acts as v_i plus a diode v-drop, (of approximately 0.65V), when v_o is positive and as v_i minus a diode v-drop when v_o is negative. Fig. 2 shows the waveforms for the oscillator with nonzero input voltage, $v_i = 0.85\text{V}$.

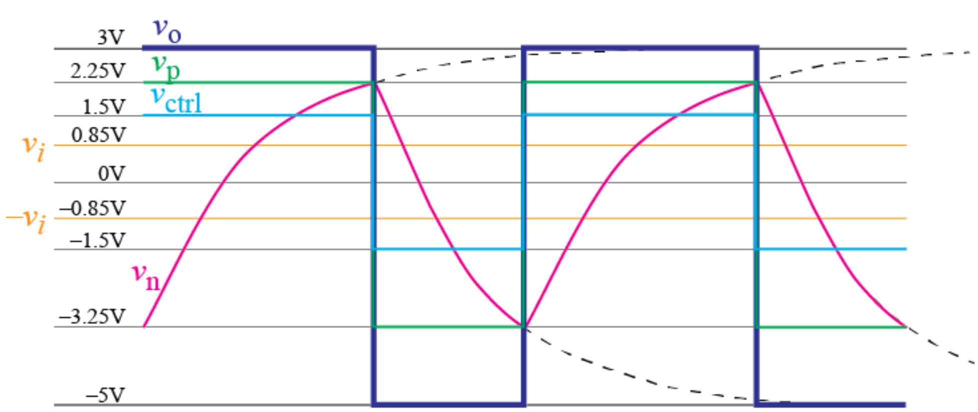

Fig. 2. Oscillator waveforms for LM324 op-amp [1] with ±5V supplies, $R_a = R_b$, and $v_i = 0.85\text{V}$.

Fig. 3 shows the waveforms for the oscillator with input voltage $v_i = 0\text{V}$.

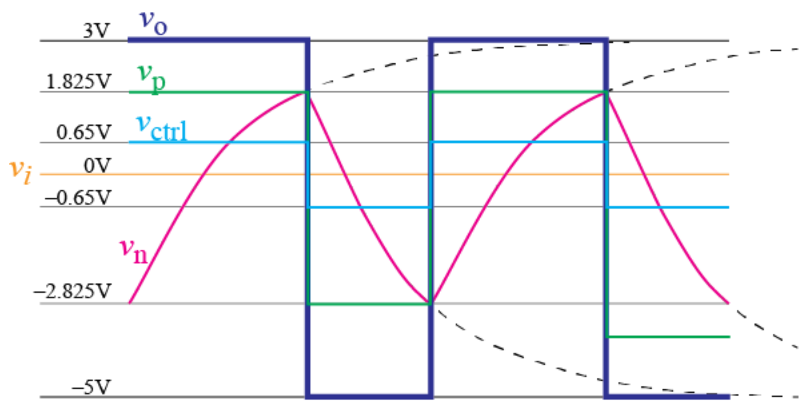

Fig. 3. Oscillator waveforms for LM324 op-amp [1] with ±5V supplies, $R_a = R_b$, and $v_i = 0\text{V}$, [1].

When the output goes high, the capacitor starts charging toward the positive rail voltage. The positive rail voltage, along with potentiometer, R_a, R_b, and the control voltage, v_{ctrl}, create a voltage divider that determines how high the output voltage, v_o, rises before the op-amp, acting as a comparator, switches to negative rail voltage output. The same voltage divider is now fed by a negative voltage that determines how low the output voltage, v_o, drops before the op-amp, acting as a comparator, switches to positive rail voltage output. The cycle then repeats.

Analysis of circuit:

To determine the timing of the output waveform, we solve *RC* charging problems for the rising capacitor voltage. The solution for falling capacitor voltage is obtained by switching the v_{+rail} and v_{-rail} and inverting the value of v_{ctrl}.

The initial voltage for the *RC* charging problems is the trip point, v_p in Fig. 2, determined by the voltage divider fed by v_o and v_{ctrl}.

$$v_C(0^-) = v_p(0^-) = \frac{v_{-rail}R_b + v_{ctrl}R_a}{R_a + R_b}$$

The final destination voltage is the positive rail voltage for v_o, although switching occurs before this voltage is reached.

$$v_C(t \to \infty) = v_{+rail}$$

The time constant, RC, primarily determines the oscillation frequency, whereas v_{ctrl} primarily controls the duty cycle.

$$\tau = RC$$

It is recommended that the duty cycle be set first with v_{ctrl}. Then R may be adjusted to set the oscillation frequency.

The equation for the charging and discharging curves:

$$v_C(t) = \left[v_C(0^-) - v_C(t \to \infty) \right] e^{-t/\tau} + v_C(t \to \infty).$$

Solving for the time of a half-cycle:

$$v_C(t) = v_{\text{p}} = \frac{v_{+\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} = \left[v_C(0^-) - v_C(t \to \infty) \right] e^{-t/\tau} + v_C(t \to \infty)$$

or

$$v_C(t) = \frac{v_{+\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} = \left[\frac{v_{-\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} - v_{+\text{rail}} \right] e^{-t/\tau} + v_{+\text{rail}}$$

or

$$\ln\left(\frac{\dfrac{v_{+\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} - v_{+\text{rail}}}{\dfrac{v_{-\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} - v_{+\text{rail}}} \right) = -t/\tau$$

or

$$t = -\tau \ln\left(\frac{\dfrac{v_{+\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} - v_{+\text{rail}}}{\dfrac{v_{-\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} - v_{+\text{rail}}} \right) = \tau \ln\left(\frac{\dfrac{v_{-\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} - v_{+\text{rail}}}{\dfrac{v_{+\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}}}{R_{\text{a}} + R_{\text{b}}} - v_{+\text{rail}}} \right)$$

or

$$t = \tau \ln\left(\frac{v_{-\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}} - v_{+\text{rail}}(R_{\text{a}} + R_{\text{b}})}{v_{+\text{rail}} R_{\text{b}} + v_{\text{ctrl}} R_{\text{a}} - v_{+\text{rail}}(R_{\text{a}} + R_{\text{b}})} \right)$$

or

$$t = \tau \ln\left(\frac{v_{-\text{rail}}\dfrac{R_\text{b}}{R_\text{a}} + v_\text{ctrl} - v_{+\text{rail}}(1+\dfrac{R_\text{b}}{R_\text{a}})}{v_\text{ctrl} - v_{+\text{rail}}} \right) = \tau \ln\left(\frac{v_\text{ctrl} - v_{+\text{rail}}(1+\dfrac{R_\text{b}}{R_\text{a}}) + v_{-\text{rail}}\dfrac{R_\text{b}}{R_\text{a}}}{v_\text{ctrl} - v_{+\text{rail}}} \right)$$

or, reversing signs in the numerator and denominator,

$$t = \tau \ln\left(\frac{v_{+\text{rail}} - v_\text{ctrl} + (v_{+\text{rail}} - v_{-\text{rail}})\dfrac{R_\text{b}}{R_\text{a}}}{v_{+\text{rail}} - v_\text{ctrl}} \right) = \tau \ln\left(1 + \frac{v_{+\text{rail}} - v_{-\text{rail}}}{v_{+\text{rail}} - v_\text{ctrl}} \cdot \frac{R_\text{b}}{R_\text{a}} \right).$$

For $R_\text{a} = R_\text{b}$, as in Figs. 2 and 3,

$$t = \tau \ln\left(1 + \frac{v_{+\text{rail}} - v_{-\text{rail}}}{v_{+\text{rail}} - v_\text{ctrl}} \right).$$

For the waveforms in Figs. 2 and 3, we have the following difference of rail voltages:

$$v_{+\text{rail}} - v_{-\text{rail}} = 3 - (-5)\,\text{V} = 8\,\text{V}.$$

For the waveforms in Fig. 2, we have the following calculation:

$$t = \tau \ln\left(1 + \frac{8\,\text{V}}{3 - 1.5\,\text{V}} \right) = \tau \ln\left(1 + \frac{8}{1.5} \right) \approx \tau \cdot 1.85 \text{ for output high,}$$

and

$$t = \tau \ln\left(1 + \frac{-8\,\text{V}}{-5 - -1.5\,\text{V}} \right) = \tau \ln\left(1 + \frac{8}{3.5} \right) \approx \tau \cdot 1.2 \text{ for output low.}$$

For the waveforms in Fig. 3, we have the following calculations:

$$t = \tau \ln\left(1 + \frac{8\,\text{V}}{3 - 0.65\,\text{V}} \right) = \tau \ln\left(1 + \frac{8}{2.35} \right) \approx \tau \cdot 1.5 \text{ for output high,}$$

and

$$t = \tau \ln\left(1 + \frac{-8\,\text{V}}{-5 - -0.65\,\text{V}} \right) = \tau \ln\left(1 + \frac{8}{4.35} \right) \approx \tau \cdot 1.05 \text{ for output low.}$$

Note that the change in timing is on the order of 20%, which is modest, for the examples given.

NOTE: The v_i control block could be modified to add an offset to the output voltages in order to produce a square wave.

REF: [1] https://www.fairchildsemi.com/datasheets/1N/1N914.pdf (accessed 23 July 2017)