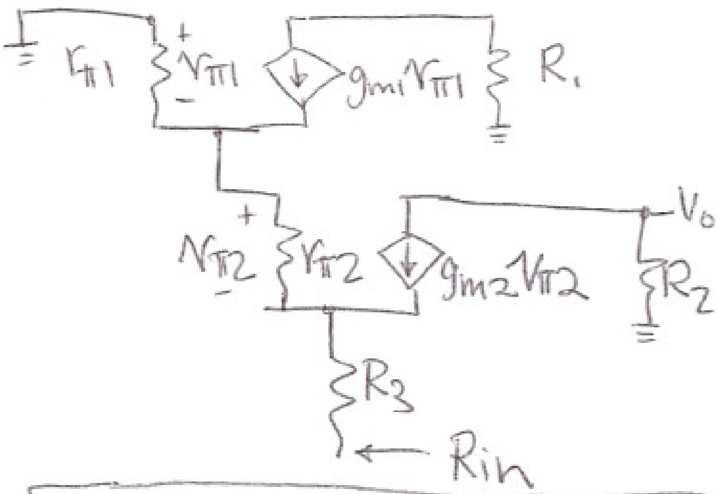
HW #5 sol'n

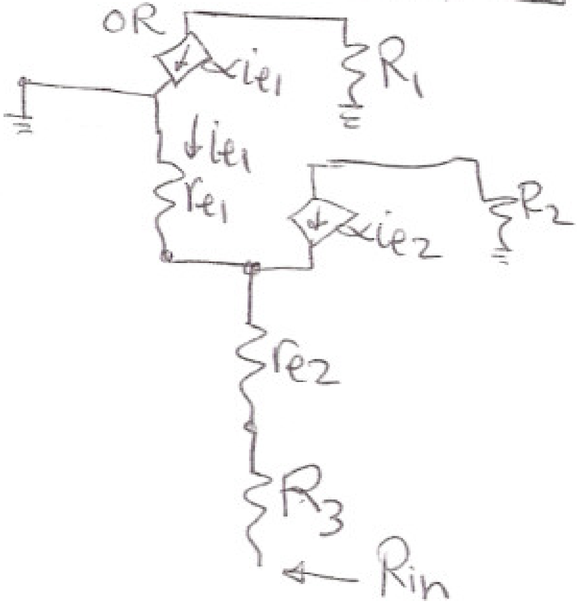
1.



$r_e(\beta+1) = r_\pi$

$$\boxed{R_{in} = R_3 + \frac{r_{\pi 2}}{(\beta+1)} + \frac{r_{\pi 1}}{(\beta+1)^2}}$$

OR



$$\boxed{R_{in} = R_3 + r_{e2} + \frac{r_{e1}}{(\beta+1)}}$$

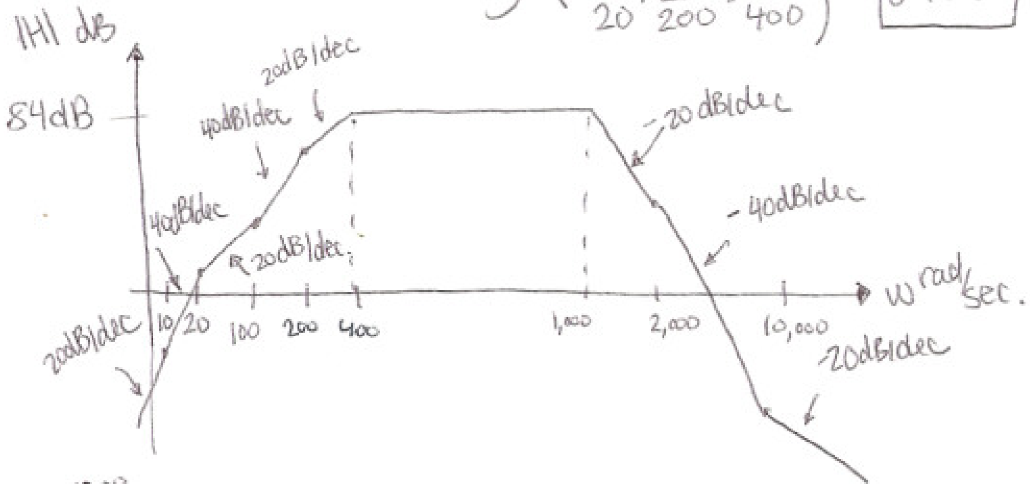
2. $$H(s) = \frac{10s\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)\left(\frac{s}{10000}+1\right)}{\left(\frac{s}{20}+1\right)\left(\frac{s}{200}+1\right)\left(\frac{s}{400}+1\right)\left(\frac{s}{1,000}+1\right)\left(\frac{s}{2,000}+1\right)}$$

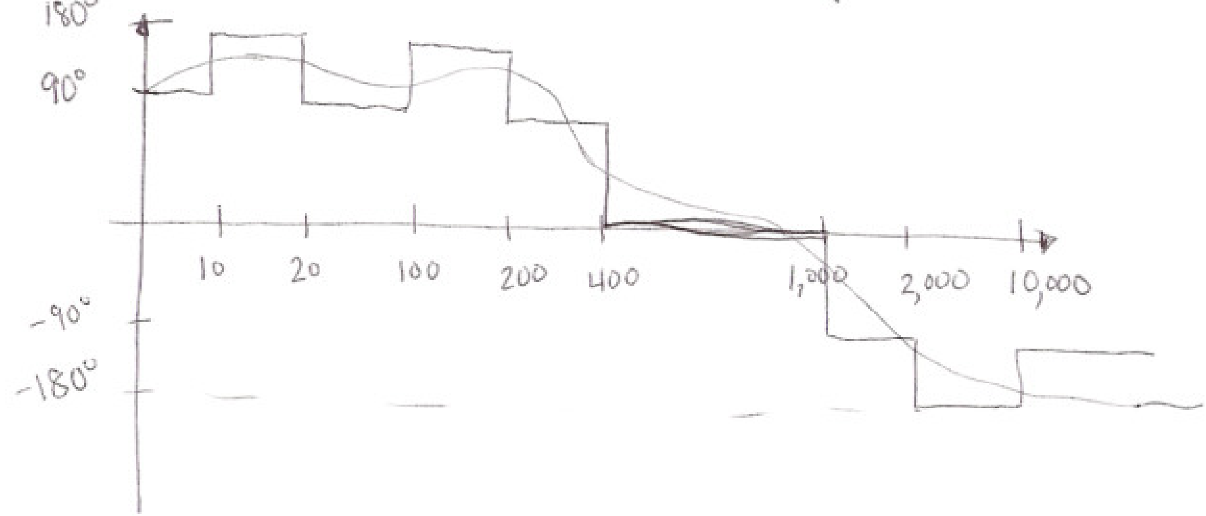
pole frequencies = 20, 200, 400, 1,000, 2,000 rad/sec.

zero frequencies = 1 at origin, 10, 100, 10,000 rad/sec.

$$H(s) = \frac{10 \cdot \cancel{s} \cdot \frac{\cancel{s}}{10}\left(1+\frac{10}{s}\right) \cdot \frac{\cancel{s}}{100}\left(1+\frac{100}{s}\right)\left(\frac{s}{10,000}+1\right)}{\frac{\cancel{s}}{20}\left(1+\frac{20}{s}\right) \cdot \frac{s}{200}\left(1+\frac{200}{s}\right) \cdot \frac{\cancel{s}}{400}\left(1+\frac{400}{s}\right)\left(\frac{s}{1,000}+1\right)\left(\frac{s}{2,000}+1\right)}$$

midband gain = $20 \log \left(\frac{10 \cdot \frac{1}{10} \cdot \frac{1}{100}}{\frac{1}{20} \cdot \frac{1}{200} \cdot \frac{1}{400}} \right) \cong \boxed{84dB}$

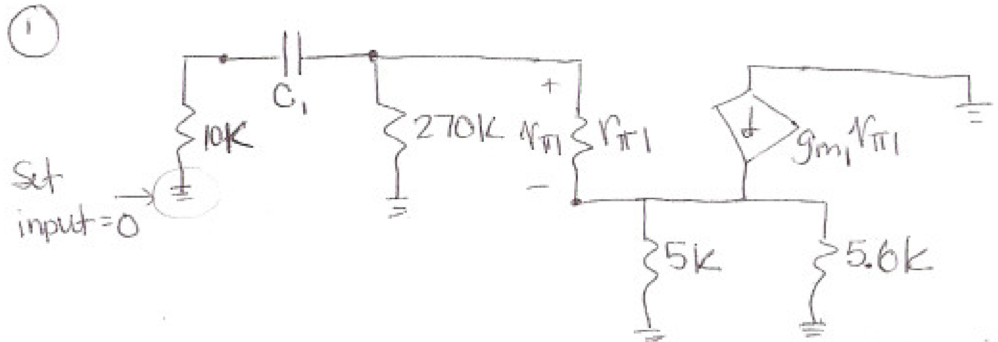




3. C_1 and C_2 affect the low frequency:

· Using short-circuit time constants ⇒

①



Resistance seen by C_1 ⇒ {Using Resistance reflection rule}

$$10K + [270K \,||\, \{(5K||5.6K)(\beta+1) + r_{\pi 1}\}]$$

$$r_{\pi 1} = \frac{V_T \beta}{I_C} = \frac{V_T}{I_B}$$

$$I_B \Rightarrow -5 + 270K(I_B) + 0.7 + 5K(I_E) = 0$$

$$I_B(\beta+1) = I_E$$

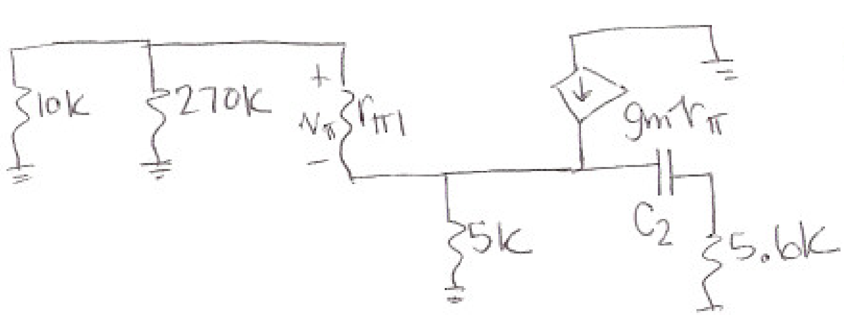
$$+4.3 = [270K + 5K(\beta+1)]\, I_B$$

$$\therefore \quad I_B = 4.9\mu$$

$$r_{\pi 1} = \frac{25m}{4.9\mu} = 5102$$

$$R_{c_1} = 10K + [270K \,||\, 324{,}725] = 157{,}422$$

②



$$R_{c_2} = 5.6k + \Big[5k \,\Big|\Big|\, \underbrace{\frac{(270K||10K) + r_{\pi 1}}{\beta+1}}_{122}\Big]$$

$$R_{c_2} = 5{,}719$$

3. $$\omega = 2\pi(100) = \frac{1}{C_1 \cdot R_{c1}} + \frac{1}{C_2 \cdot R_{c2}}$$

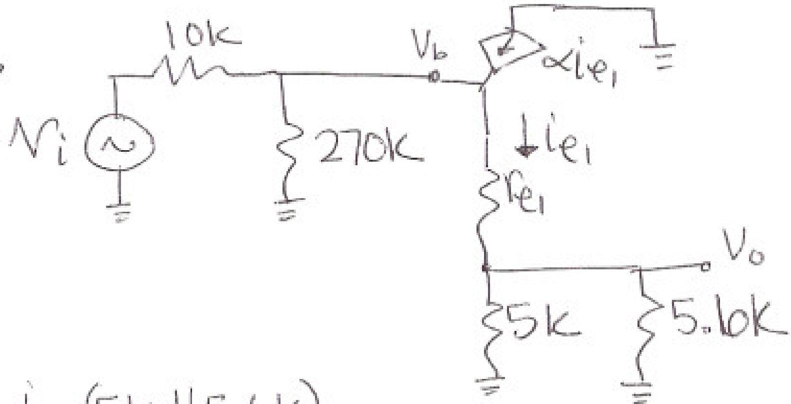
$$\left[628 = \frac{1}{10C_2 \cdot 157{,}422} + \frac{1}{C_2 \cdot 5{,}719}\right] C_2$$

$$C_2 = \frac{\left[\frac{1}{10(157{,}422)} + \frac{1}{5{,}719}\right]}{628} \cong \underline{279\text{nF}}$$

$$\therefore \; C_1 = \underline{2.79\mu\text{F}}$$

midband gain, A_m ⇒ · same as overall gain (short caps)

$\frac{V_o}{V_i}$ ⇒



$$V_o = i_{e1}(5k \,||\, 5.6k)$$

$$i_{e1} = \frac{V_b}{r_{e1} + (5k \,||\, 5.6k)} \qquad V_b = \frac{V_i(270k)}{270k + 10k}$$

$$r_{e1} = \frac{r_{\pi 1}}{(\beta+1)} = \frac{5102}{121} \cong 42.2$$

$$\frac{V_o}{V_i} = \frac{(270k)}{(280k)} \cdot \frac{1}{(42.2 + 2642)}(2642) = 0.95$$

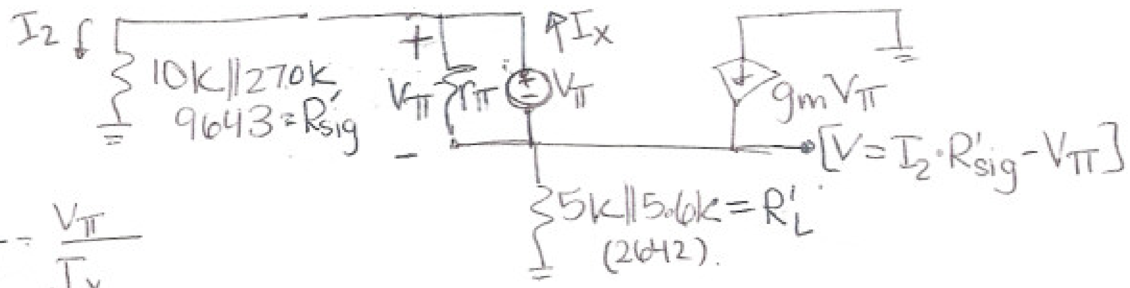
$$A_m \text{ in dB} = \boxed{-0.45\text{dB or } 0.95 \, V/V}$$

3. $\omega_L = \boxed{628 \text{ rad/sec or } 100\text{Hz}}$

high-frequency $\Rightarrow$ · use open-circuit time constants

$$\omega_H \approx \frac{1}{C_\mu R_\mu + C_\pi R_\pi}$$

C_π: (short bypass/coupling caps, set input = 0)



$$R_{c\pi} = \frac{V_\pi}{I_x}$$

$$I_x - \frac{V_\pi}{r_\pi} - I_2 = 0 \Rightarrow I_2 = I_x - \frac{V_\pi}{r_\pi}$$

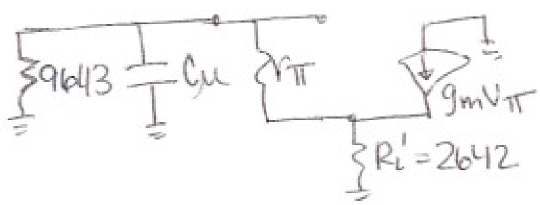
$$g_m V_\pi - I_x + \frac{V_\pi}{r_\pi} - \frac{V}{R'_L} = 0 \Rightarrow \left(g_m + \frac{1}{r_\pi}\right)V_\pi - I_x - \frac{\left[\left(I_x - \frac{V_\pi}{r_\pi}\right)R'_{sig} - V_\pi\right]}{R'_L} = 0$$

$$g_m = \frac{\beta}{r_\pi}, \text{ so } \left(g_m + \frac{1}{r_\pi}\right) = \left(\frac{\beta+1}{r_\pi}\right) = \frac{1}{r_e}$$

$$\frac{V_\pi}{I_x}\left(\frac{1}{r_e} + \frac{1}{r_\pi R'_L} + \frac{1}{R'_L}\right) = \cancel{I_x}\left(1 + \frac{R'_{sig}}{R'_L}\right)$$

$$R_{c\pi} = \frac{V_\pi}{I_x} = \frac{\left(1 + \frac{R'_{sig}}{R'_L}\right)}{\left(\frac{1}{r_e} + \frac{R'_{sig}}{r_\pi R'_L} + \frac{1}{R'_L}\right)} \cdot \frac{R'_L}{R'_L} = \frac{(R'_L + R'_{sig})}{\left(\frac{R'_L}{r_e} + \frac{R'_{sig}}{r_\pi} + 1\right)} = \frac{2642 + 9643}{\frac{2642}{42.2} + \frac{9643}{5102} + 1} \cong 188$$

C_μ:



$$R_{c\mu} = 9643 \parallel [\underbrace{2642(\beta+1) + r_\pi}_{324{,}784}] = 9365$$

$$\omega_H \approx \frac{1}{(0.5p)(9365) + (0.2p)(188)} \cong \boxed{219\text{M rad/sec.}}$$

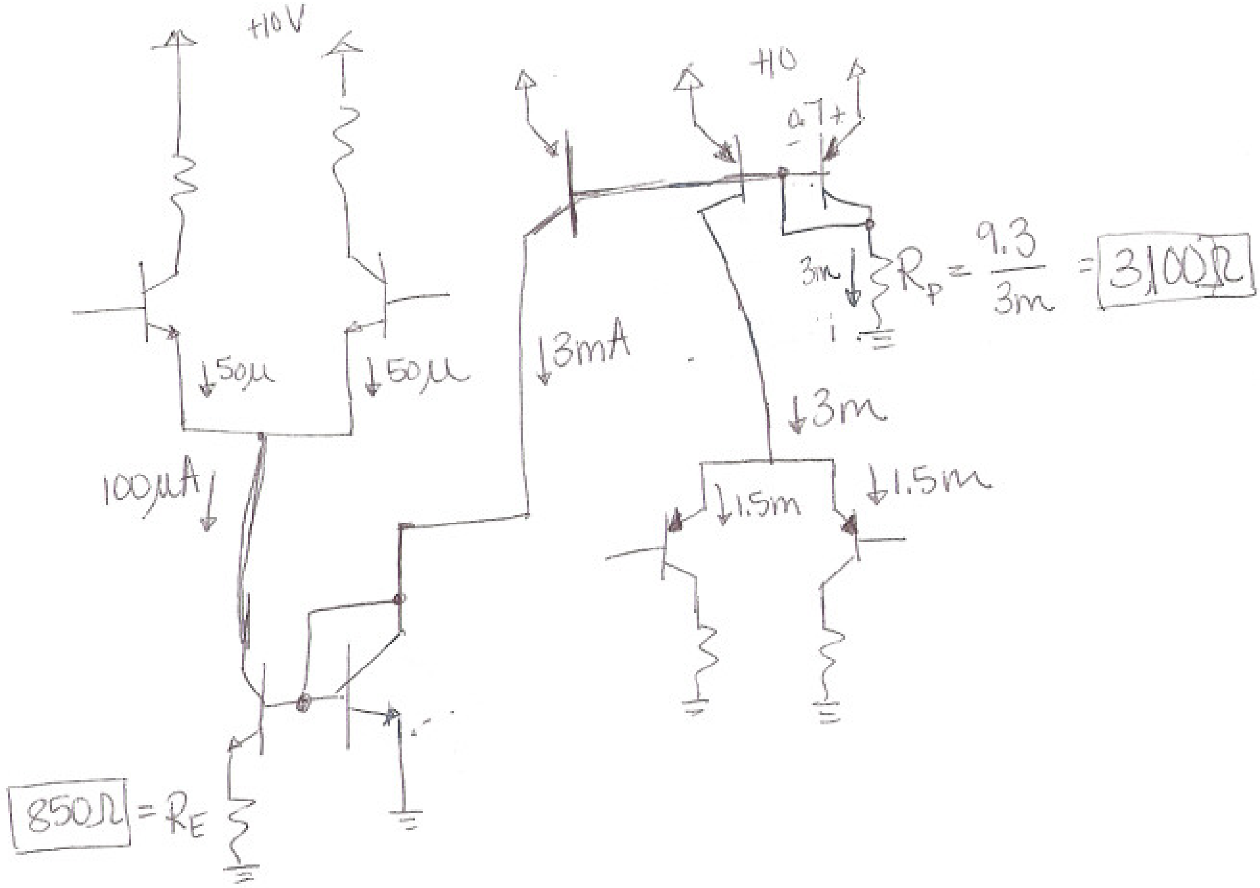
or $\underline{33.7 \text{ MHz}}$

4. NPN $\Rightarrow$ $g_m = 2m = \frac{I_{c_n}}{25m} \Rightarrow I_{c_n} = 50\mu A$

$\therefore$ $I_1 = 2 \cdot I_{c_n} = \underline{100\mu A}$

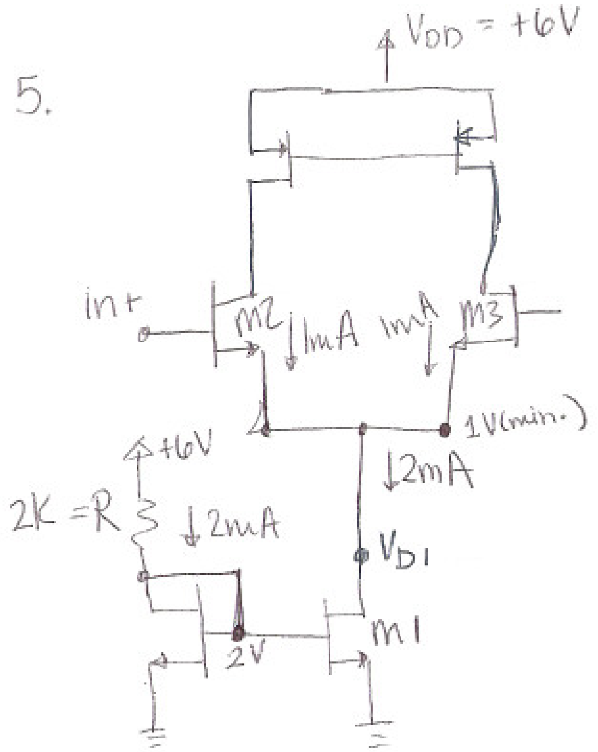
PNP $\Rightarrow$ $g_m = 60 mA/V = \frac{I_{cp}}{25m} = 1.5mA$

$\therefore$ $I_2 = 2 \cdot I_{cp} = \underline{3mA}$



$$R_E = \frac{V_T}{100\mu} \ln\left(\frac{3m}{100\mu}\right) = 850\Omega$$

5.



$$A_d = g_m \frac{R_o}{2} \Rightarrow$$

$$g_m = \frac{2I_D}{V_{ov}}$$

$$\therefore A_d = 50 = \frac{\cancel{2I_D}}{V_{ov}} \cdot \frac{V_A}{\cancel{2I_D}} = \frac{20}{V_{ov}}$$

$$V_{ov} = 0.4$$

$$V_{GS} - V_t = 0.4$$

$$\therefore V_{GS} = \boxed{1.4V}$$

Needs to have this voltage bias on the gate to operate

Current mirror $\Rightarrow$

$$2m = \frac{1}{2}(4m)(V_{GS} - V_t)^2$$

$$1 = V_{GS} - V_t$$

$$\therefore V_{GS} = 1 + 1 = 2V$$

$$R = \frac{6-2}{2m} = \boxed{2K}$$

minimum drain voltage for M1 to be saturated $\Rightarrow$

$$V_{DS_1} = V_{GS_1} - V_{t_1}$$

$$\therefore V_{D1} = V_{G1} - V_t = 2 - 1 = 1V.$$

Since M2 & M3 need $V_{GS} = 1.4V$ to operate

$$\therefore V_G - V_S = 1.4V$$

$$(\text{min.})\ V_G = 1.4 + 1V = \boxed{2.4V}$$