Fundamentals of Digital System Design

ECE/CS 3700
Spring 2009, Homework # 1
Due Date: Fri, Jan 30, 2009
Please deposit the HW in the 3700 HW locker in MEB

Note: The following set of questions correspond to chapter 2 in the textbook. I would advice you to go through chapter 2 (omit the Verilog related section - 2.10) before solving these problems. The homework is due on Jan 30. Show all your work. Should you have difficulty in understanding any of the questions, feel free to ask the TAs or the instructor. Good luck!

1. **(Simplification using Boolean Algebra - 20 points)** Using the laws of Boolean algebra, prove (or disprove) the following:

(a) $(X + Y) \cdot (X + \overline{Y}) = X$

(b) $(X) \cdot (X + Y) = X$

(c) $(X) \cdot (X + \overline{Y}) = X$

(d) $(X + Y) \cdot (\overline{X} + Z) = XZ + \overline{X}Y$. By the way, we have studied this function in class. What is it?

(e) $X \cdot Y \cdot Z + X \cdot \overline{Y} = X \cdot \overline{Y} + X \cdot Z$

2. **(20 points)** Simplify the following expressions as much as possible:

(a) $(x + y)(\overline{x} + y)(x + \overline{y})(\overline{x} + \overline{y})$

(b) $\overline{x}(\overline{y} + \overline{z})(x + y + \overline{z})$

(c) $\overline{a}\overline{b} + \overline{a}b + a\overline{b} + ab$

(d) $\overline{A} \cdot B \cdot (\overline{D} + \overline{C}D) + B(A + \overline{A}CD)$

3. **(3-var XOR/XNOR - 20 points)** In class, we have analyzed the Exclusive-OR (XOR) function of two variables: $f(a, b)$, which is represented as $f = a \oplus b = ab' + a'b$. In this exercise, you are asked to derive a sum-of-product (SOP) form expression for a 3-variable XOR function $f(a, b, c) = a \oplus b \oplus c$. Similarly, derive a SOP form for a 3-variable XNOR function $f(a, b, c) = a\overline{\oplus}b\overline{\oplus}c$. By the way, XOR functions have many interesting properties; one of which you are kindly requested to prove (or disprove): $f(a, b) = \overline{a} \oplus b = a \oplus \overline{b} = a\overline{\oplus}b$.
Note: You should also be able to construct a truth-table for the 3-input XOR function. Try to do this yourself and see what the truth-table looks like. (Not required for the HW, but I encourage you to do this).

4. **(A Digital Design Example - 20 points)** You are asked to design the following warning circuit for your car. The warning signal W should be set to high voltage (logical 1) if: (i) the engine is running and the door is open; OR (ii)

with the engine running, somebody is sitting in the driver's seat and the belt is not fastened. Otherwise the output of the circuit is 0. The circuit should rely on the following sensors:

- Sensor from the engine (C=1 if engine is running, otherwise it is 0);
- Seat sensor (S=1 if somebody is sitting on the seat, otherwise 0);
- Door sensor (D=1 if the door is closed, otherwise 0);
- Belt sensor (B=1 if it the belt is fastened, otherwise 0).

Derive the truth table corresonding to the above specifications. Subsequently, derive a simplified Boolean expression and draw the logic circuit using AND, OR and NOT gates.

5. **(Minterms and Maxterms - 20 points)** Consider the Boolean function represented by the truth table shown in Table I.

TABLE I
TRUTH TABLE

x	y	z	f
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

(a) Derive the Minterm Canonical form (also called the Canonical Sum-of-Product form) expression of the Boolean function from the truth table.

(b) Simplify the derived minterm canonical form as much as possible.

(c) Derive the Maxterm Canonical form (also called the Canonical Product-of-Sum form) expression of the Boolean function from the truth table.

(d) Is the minterm canonical form of the function logically equivalent to its maxterm canonical form? If yes, prove the equivalence of the expressions. If not, prove otherwise.