ECE/CS 3700
Solutions to HW 1

Problem 1:

a)

$(X+Y)(X+Y') = XX + XY' + XY + YY'$
$= X + XY' + XY + 0$
$= X(1 + Y' + Y)$
$= X(1 + Y' + Y)$
$= X(1 + 1)$
$= X$

b)

$X(X + Y) = XX + XY$
$= X(1 + Y)$
$= X(1)$
$= X$

c)

$X(X + Y') = XX + XY'$
$= X(1 + Y')$
$= X(1)$
$= X$

d)

$(X + Y)(X' + Z) = XX' + XZ + YX' + YZ$
$= XZ + YX' + YZ(X + X')$
$= XZ + YX' + XYZ + X'YZ$
$= XZ + XYZ + YX' + X'YZ$
$= XZ(1 + Y) + YX'(1 + Z)$
$= XZ + YX'$ MULTIPLEXER FUNCTION

e)

$XYZ + XY' = X(YZ + Y')$
$= X(YZ + Y'(Z+Z'))$
$= X(YZ + Y'Z + Y'Z')$
$= X(YZ + Y'Z + Y'Z + Y'Z')$ DUPLICATE Y'Z
$= X(Z + Y')$
$= XZ + XY'$

Problem 2:

a)

$(x + y)(x' + y)(x + y')(x' + y')$
$= (x + y)(x' + y')(x' + y)(x + y')$
$= (xx' + xy' + yx' + yy')(x'x + x'y' + yx + yy')$
$= (xy' + yx')(x'y' + yx) = xy'x'y' + xy'yx + yx'x'y' + yx'yx$
$= 0y' + 0x + 0x' + 0y = 0$

b)

$x'(y' + z')(x + y + z') = (x'y' + x'z')(x + y + z')$

$= x'y'x + x'y'y + x'y'z' + x'z'x + x'z'y + x'z'z'$

$= 0y' + 0x' + x'y'z' + 0z' + x'z'y + x'z' = x'z'(y + y') + x'z'$

$= x'z'$

c)

$a'b' + a'b + ab' + ab = a'(b' + b) + a(b' + b)$

$= a' + a$

$= 1$

d)

$a'b(d' + c'd) + b(a + a'cd)$

$= a'bd' + a'bc'd + ba + ba'cd$

$= a'bd' + ba + a'bc'd + ba'cd$

$= a'bd' + ba + a'bd(c' + c)$

$= a'bd' + ba + a'bd$

$= a'b(d' + d) + ba$

$= a'b + ba$

$= b(a' + a)$

$= b$

Problem 3:

a)

$Suppose: f(a,b) = a \oplus b = ab' + a'b$

$$f(a,b,c) = a \oplus b \oplus c$$
$$f(a,b,c) = (ab' + a'b) \oplus c$$
$$f(a,b,c) = (ab' + a'b)c' + (ab' + a'b)'c$$
$$f(a,b,c) = ab'c' + a'bc' + (a' + b)(a + b')c$$
$$f(a,b,c) = ab'c' + a'bc' + a'ac + a'b'c + bac + bb'c$$
$$f(a,b,c) = ab'c' + a'bc' + a'b'c + abc$$

b)

$$f(a,b,c) = a \overline{\oplus} b \overline{\oplus} c$$
$$f(a,b,c) = (ab' + a'b)' \overline{\oplus} c$$
$$f(a,b,c) = (a' + b)(a + b') \overline{\oplus} c$$
$$f(a,b,c) = (a'b' + ab) \overline{\oplus} c$$
$$f(a,b,c) = ((a'b' + ab)c' + (a'b' + ab)'c)'$$
$$f(a,b,c) = (a'b'c' + abc' + (a + b)(a' + b')c)'$$
$$f(a,b,c) = (a'b'c' + abc' + ab'c + a'bc)'$$
$$f(a,b,c) = ab'c' + a'bc' + a'b'c + abc$$

c)

$$a'\oplus b = a'b' + (a')'b = \underline{\underline{a'b' + ab}}$$

$$a \oplus b' = a(b')' + a'b' = \underline{\underline{ab + a'b'}}$$

$$a \overline{\oplus} b = (ab' + a'b)' = (a' + b)(a + b') = a'a + a'b' + ab + bb' = \underline{\underline{ab + a'b'}}$$

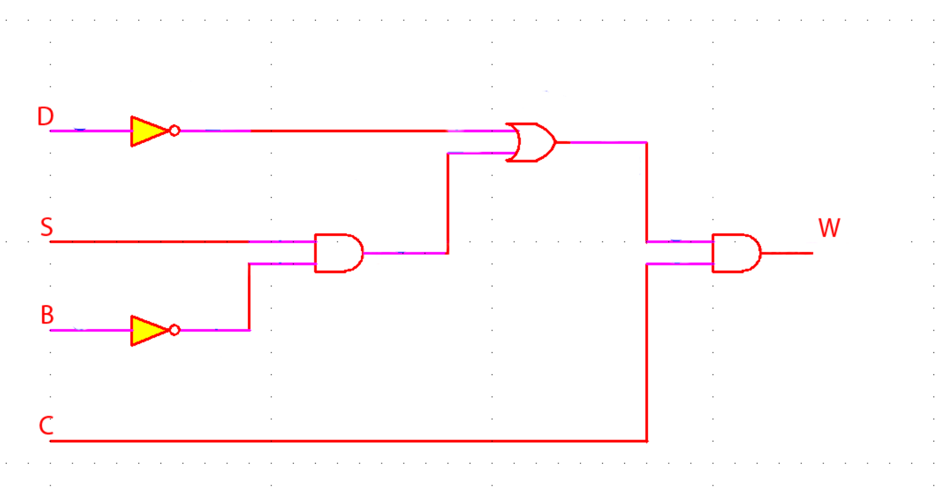
thus

$$\underline{\underline{f(a,b) = a'\oplus b = a \oplus b' = a \overline{\oplus} b}}$$

Problem 4:

C	S	D	B	W
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$$\begin{aligned}
W &= CS'D'B' + CS'D'B + CSD'B' + CSD'B + CSDB' \\
&= CS'D' + CS(D'B' + D'B + DB') \\
&= CS'D' + CS(D' + B') \\
&= C(S'D' + SD' + SB') \\
&= \underline{\underline{C(D' + SB')}}
\end{aligned}$$



Problem 5:

Truth table:

x	y	z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

a)

$f = x'y'z' + x'yz' + x'yz + xyz' + xyz$

b)

$f = x'z'(y'+y) + yz(x+x')xyz'$

$= x'z' + yz + xyz' + x'yz'$

$= x'z' + yz + yz'(x+x')$

$= x'z' + yz + yz'$

$= \underline{\underline{x'z' + y}}$

c)

$f = (x+y+z')(x'+y+z)(x'+y+z')$

d)

$f = (x+y+z')((x'+y) + z'(x'+y) + z(x'+y) + z(x'+y) + zz')$

$= (x+y+z')((x'+y) + (x'+y)(z+z'))$

$= (x+y+z')(x'+y)$

$= xx' + xy + yx' + yy + z'x' + z'y$

$= xy + xy + y + x'z' + yz'$

$= y(x + x' + 1 + z') + x'z'$

$= \underline{\underline{y + x'z'}}$