Two-Level Logic Minimization: Terminology and Problem Statement

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• **Binary Variable** = symbol. Represents a co-ordinate of Boolean space spanned by $n$-variables (called $B^n$), where $n =$ the number of variables of the function

• **Literal**: Boolean variable, or its complement

• $f = a + a'b$ has how many literals? 2 or 3?

• **Minterm**: a point in the Boolean space
  - A product of all $n$ literals

• **Cube**: a point, or a set of points in $B^n$
  - A product of literals, may contain fewer than $n$ literals

• Terminology comes from $n$-D cube

• $a'bc + abc$: 2 cubes. $bc$ is a smaller or larger cube?
n-Dimensional Cube

- Expand the cubes, 1-hamming distance at a time.
- Hamming distance?

1-Hamming distance

only 1-bit changes
Implicant: Same thing as an ON-SET cube; “implies” the value of the function (= 1)

Prime Implicant: Not contained in any other implicant

Prime implicant cannot be expanded

Prime implicant is a largest cube

One solution for logic minimization: $F = \text{all prime implicants}$

Problem: Redundancy! Too many ($\leq 3^n/n$) primes

Still have to make choices...

Greedy strategy does not always work

Quine-McCluskey gave a systematic solution to find a minimum cost cover of a function
Prime Cover: A Cover containing only prime implicants

Quine’s Theorem:
- There exists a minimum cover that is prime!
- That’s why, analyze only prime implicants
  - Quickly generate all prime implicants: Expand all ON-set cubes as much as possible!
  - Identify all essential primes
  - Now select a minimum from the remaining ones...

“Minimum number of primes” versus “A minimum number of primes with minimum cost”. See Fig. 2.57.

A Minimum Cost cover is NOT unique, see Fig. 2.54 (iv)
So, the strongest problem formulation is: \textit{Find a minimum cost cover from among the prime implicants that contains a minimum number of primes!}