Two-Level Logic Minimization: Terminology and Problem Statement

Priyank Kalla

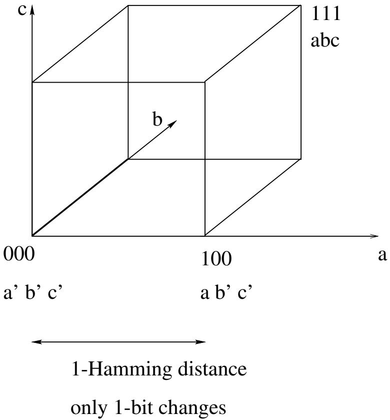
Associate Professor
Electrical and Computer Engineering, University of Utah
kalla@ece.utah.edu
http://www.ece.utah.edu/~kalla

Terminilogy

- **Binary Variable** = symbol. Represents a co-ordinate of Boolean space spanned by n-variables (called B^n), where $n =$ the number of variables of the function
- **Literal**: Boolean variable, or its complement
- $f = a + a'b$ has how many literals? 2 or 3?
- **Minterm**: a point in the Boolean space
  - A product of all n literals
- **Cube**: a point, or a set of points in B^n
  - A product of literals, may contain fewer than n literals
- Terminology comes from n-D cube
- $a'bc + abc$: 2 cubes. bc is a smaller or larger cube?

n-Dimensional Cube

- Expand the cubes, 1-hamming distance at a time.
- Hamming distance?



Implicants of a Function

- Implicant: Same thing as an ON-SET cube; "implies" the value of the function ($= 1$)
- **Prime Implicant:** Not contained in any other implicant
- Prime implicant cannot be expanded
- Prime implicant is a largest cube
- One solution for logic minimization: $F =$ all prime implicants
- Problem: Redundancy! Too many ($\leq 3^n/n$) primes
- Still have to make choices...
- Greedy strategy does not always work
- Quine-McCluskey gave a systematic solution to find a **minimum cost cover of a function**

Exact Logic Minimization

- *Prime Cover:* A Cover containing only prime implicants
- **Quine's Theorem:**
  - There exists a minimum cover that is prime!
- Thats why, analyze only prime implicants
  - Quickly generate all prime implicants: Expand all ON-set cubes as much as possible!
  - Identify all essential primes
  - Now select a minimum from the remaining ones...
- "Minimum number of primes" versus "A minimum number of primes with minimum cost". See Fig. 2.57.
- A Minimum Cost cover is NOT unique, see Fig. 2.54 (iv)

So, the strongest problem formulation is: *Find a minimum cost cover from among the prime implicants that contains a minimum number of primes!*