HW6 solutions

Q1 P.7, ch11.

(a) — SDC are straightforward

$$SDC\, y_2 = y_2 \oplus ac \quad \text{and so on.....}$$

(b) $ODC\, y_4 = (y_5 = 0 \;\&\; y_2 = 1)$

$$= \bar{y}_5 + y_2$$

$$= ab + ac$$

$$ODC\, y_5 = y_4 = 0,\; y_2 = 1$$

$$= \bar{y}_4 + y_2$$

$$= \overline{a\bar{c}} + ac = \bar{a} + c + ac = \bar{a} + c$$

(c) $F_4 = a\bar{c}$

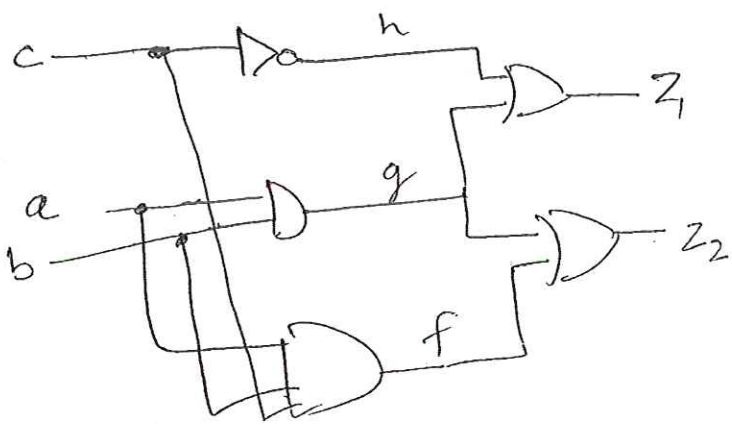
$$F_4 + ODC\, y_4 = a\bar{c} + \underbrace{\cancel{ab}}_{\text{Don't care}} + ac = \underline{\underline{a}}$$

$$F_5 = \overline{ab} = \bar{a} + \bar{b}$$

$$F_5 + ODC\, y_5 = \underbrace{\bar{a} + \bar{b} + \bar{a} + c}_{D.C}$$

$$= \bar{b}$$

Q2



ODC h $\equiv g=1 = ab$

ODC g @ Z_1 $\equiv h=1 = \bar{c}$

ODC g @ Z_2 $\equiv f=1 = abc$

ODC f @ Z_2 $\equiv g=1 = ab.$

Simplify h w/ ODC $\quad h + ODC_h$

$= \bar{c} + ab \quad = \bar{c}$

No simplification.

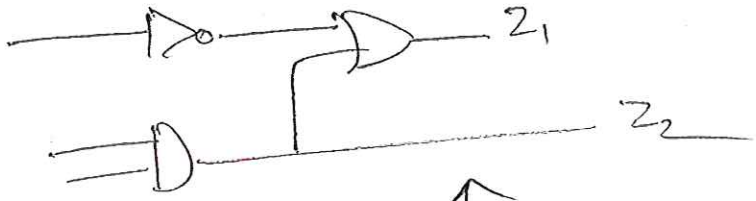
Simplify $g = ab$ w/ $\bar{c}$ or abc $\longrightarrow$ No simplification

Simplify f w/ $ODC_f = ab$

$= abc + ab = ab(c+1) = ab.$

But $ab = D.C$

So $f = 0$



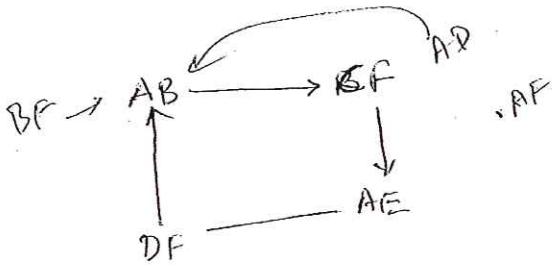
Simplified CKT

Q3

machine

M1

	A	B	C	D	E
B	CF ✓				
C	×	×			
D	FD ✓	~~CD~~	×		
E	FD ✓	×	×	×	
F	✓	AB ✓	AE ✓	AB ✓	×



Compatibles:-

AB, AD, AE, AF

BF, CF, DF

$$= \frac{(ADF)\,(AB)\,(AE)\,(BF)\,(CF)}{(AF) \quad \times \quad \times}$$

$$= \underbrace{(ADF)(ABF)(AE)(CF)}_{\text{Upper bound on \# of states}}$$

Note that the set of maximal compatibles is itself the Solution $\longrightarrow$ all of these are needed for covering

Logic Synthesis & Optimization

Spring 2008, Solutions to Homework # 6
Prepared by Priyank

1. Extra write-up for minimizing M1, and for the encoding problem.

2. M1 is an incompletely specified machine. The first thing that you need to do is to identify compatible pairs. You can use either the merger table, or the merger graph. If you build the merger table, you can see that the only compatible (BD) gets cancelled because its implied pair (CD) is not compatible.

Compatibles are: (AF)(AE)(AD)(AB)(BF)(CF)(DF). Max compatibles: (ABF) (ADF) (CF) (AE).

Note that (AE) and (CF) are the only compatibles in the maximal set that contain C and E. Therefore, they have to be in the solution. That takes care of A, C, E, F. What about B and D? They are also in different compatible groups. Clearly, all 4 have to be in the solution. Now the question is whether their implied pairs are also in the maximal compatible sets? That they are.

Note: From the maximal compatible set, we can find a minimum number of compatibles that cover the entire machine. This guarantees covering, but not closure. On the other hand, the set containing all maximal compatibles is clearly a closed covering. Therefore, this is an upper bound on the # of states. However, in general, we may be able to get a better solution - a smaller set. And thats why we need to go into that compatibility graph and all that we studied in class.

In our current example, we don't need to solve the problem on the compatibility graph. This is because, from the maximal compatible set we can figure out that we need all maximal compatibles just to cover the machine. And since the set of all maximal compatibles also guarantees closure, we can just use the set of all maximal compatibles as our solution.

However, for your benefit, I'm still solving the problem.

Q2 (M2)

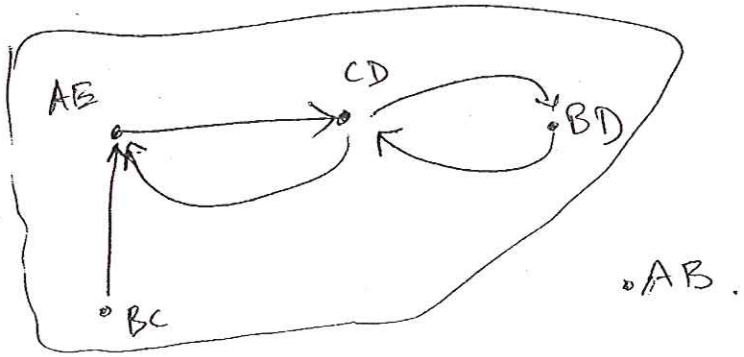
Compatible pairs:- AB, AE, BC, BD, CD

B	✓			
C	×	AE		
D	×	CD	BD, AE	
E	CD	×	×	×
	A	B	C	D

AB, AE, BCD

↓

Cover but closure? Yes!



Cover 1 :→ AE, CD, BD → 3 states.

Cover 2 :→ BC, AE, CD, BD

(BCD), (AE)

2 states.

Q4 Now let us solve M2 using BCP.

Max. comp:- BCD, AB, AE

P_1 ✓ $BCD \longrightarrow AE$

P_2 ✓ $AB \longrightarrow \phi$

P_3 ✓ $AE \longrightarrow CD$

primes

$BC \longrightarrow AE$ ✗ not prime, $\subseteq P_1$

P_4 ✓ $BD \longrightarrow CD$

$CD \longrightarrow BD, AE$ ✗ not prime, P_1 = cheaper.

$A \longrightarrow \phi$ ✗ not prime, $\subseteq P_2$

$B \longrightarrow \phi$ " "

P_5 ✓ $C \longrightarrow \phi$

P_6 ✓ $D \longrightarrow \phi$

P_7 ✓ $E \longrightarrow \phi$

State A :- $(P_2 + P_3)$

B :- $(P_1 + P_2 + P_4)$

C :- $(P_1 + P_5)$

D :- $(P_1 + P_4 + P_6)$

E :- $(P_3 + P_7)$

Cover

$P_1 \rightarrow P_3 \equiv \overline{P_1} + P_3$ C1

$P_3 \rightarrow CD \subseteq BCD$

$P_3 \rightarrow P_1 \equiv \overline{P_3} + P_1$ C2

$BD \rightarrow CD$

$P_4 \rightarrow P_1$

$\overline{P_4} + P_1$ C3

Set-up BCP for M2.

Columns = prime compatibles $P_1 \ldots\ldots P_7$

rows = ~~clos~~ state covering + closure constraints

Select a minimum # of primes (columns) s.t. all constraints (rows) are SAT.

		P_1	P_2	P_3	P_4	P_5	P_6	P_7	
Covering	A	–	1	1	–	–	–	–	(P_2+P_3)
	B	1	1	–	1	–	–	–	$(P_1+P_2+P_4)$
	C	1	–	–	–	1	–	–	(P_1+P_5)
	D	1	–	–	1	–	1	–	$(P_1+P_4+P_6)$
	E	–	–	1	–	–	–	1	(P_3+P_7)
Closure	C1	0	–	1	–	–	–	–	$(\bar{P}_1+P_3)$
	C2	1	–	0	–	–	–	–	$(\bar{P}_3+P_1)$
	C3	1	–	–	0	–	–	–	$(\bar{P}_4+P_1)$

No essentials, no dominance

$\Rightarrow$ cyclic. M

Now compute MIS.

ignore rows with zeros (C_1, C_2, C_3)

$$L.B = MIS = \left\{ \begin{matrix} \text{row } A \\ \& \\ \text{row } C \end{matrix} \overset{=2}{} \text{ or } \begin{matrix} \text{row } B \\ \& \\ \text{row } E \end{matrix} \overset{=2}{} \right\} = \underline{\underline{2}}$$

maximally independent set of constraints.

Branch on P_1 (greedy, most entries)

$P_1 = 1$

	P_2	P_3	P_7
A	1	1	—
E	—	1	1
C1	—	1	—

P_3 = essential

$P_1 + P_3 = \underline{L.B = 2}$

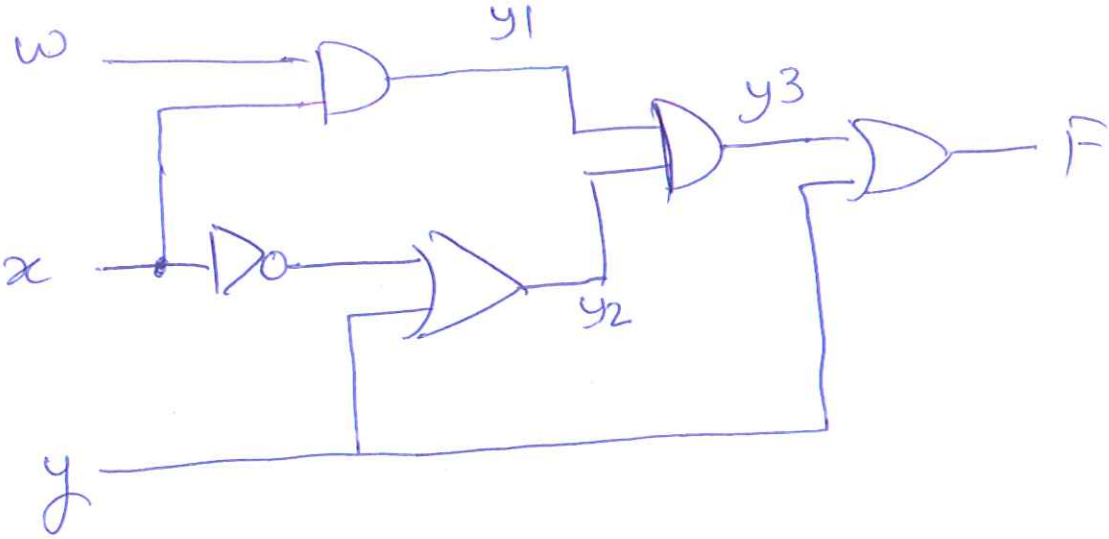
Stop.

P_1 & P_3 = solution

(BCD) (AE)

2 state M/C.

Q3 Static false paths:



Path 1: $w \to y_1 \to y_3 \to F$.

2: $x \to y_1 \to y_3 \to F$

3: $x \to y_2 \to y_3 \to F$

4: $y \to y_2 \to y_3 \to F$.

5: $y \longrightarrow F$.

P_1 = Propagate w to F require $x=1$ & $y_2=1$ & $y=0$.

not possible.

P_1 = false

statically, $w/1$ and $w/0$ not testable.

put $w=1$ & simplify or $w=0$ and simplify.

better $0 \Rightarrow \overline{0}$

$y_1 = 0$.

$F = y$ (beautiful ckt!)

Path p_2:

needs $y_2 = 0$. & $w = 1$

$\Rightarrow \quad x \cdot \bar{x}$ @ $y_3 = 0$.

also false.

S/0 @ x $\neq$ testable.

Put $x = 0$ & simplify. $F = y$!

No surprise that P_3 = false too.

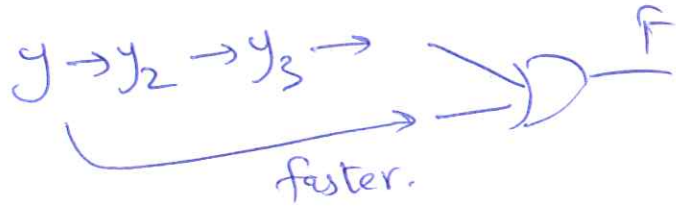
~~P_3 and P_4 also false~~.

P_4? $\quad y \rightarrow y_2$ requires $x = 1$

$y_2 \rightarrow y_3$ requires $y_1 = 1 = x = w = 1$

statically, P_4 not false.

But

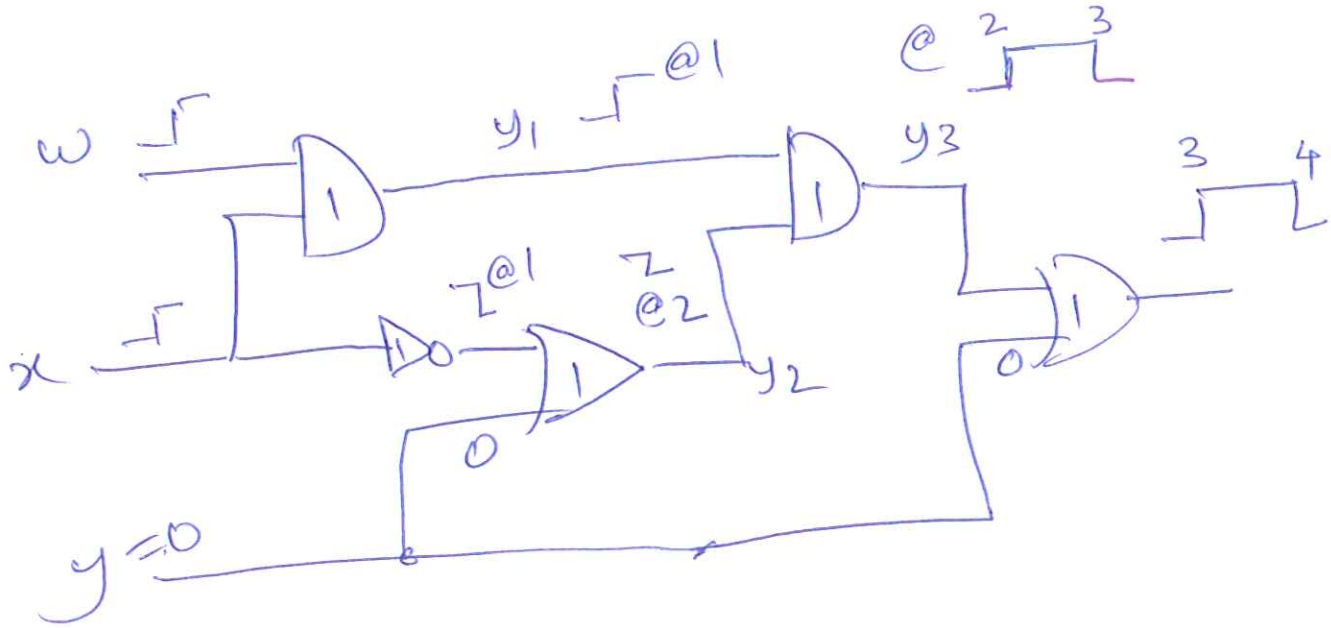


So $y \rightarrow y_2 \rightarrow y_3 \rightarrow F$ has no effect, so it is actually false if timing were to be analyzed.

Static sensitization $\not\Longrightarrow$ false paths.

Now the other way:

P_1 = is it truely timing false?



P_1 controlled by x, also reconvergent @ y_3 with x ⇒ potential problem.

Assume gate delays = 1

$w =$ ⎍ rising transition.

$x =$ ⎍ " "

$y = 0$

glitches flowing through P_1 & P_2 & P_3

So they are <u>not</u> "really false".