Retiming

- Operate on a synchronous network:
  - Propagation delays of the nodes
  - Wts. On edges = # registers (called path weights)
- Retiming of a network $G(V, E, W) = G'(V, E, W')$
- Retiming across a node ($v_i$) = $r_i$ = +ve (back), -ve (forw.)
- Weight after retiming: $w_{ij}' = w_{ij} + r_j - r_i$ (extremal vert.)
- Init: $w_{ab} = 0$, retime across a, $r_a = -1$, $r_b = 0$, $w_{ab}' = 0 + 0 - (-1)$
- $W_{ad}' = w_{ad} + r_d - r_a = -1 - (-1) = 0$ (no regs. Between a-d)
- $W_{ac}'$ (after retiming from a to d) =

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A -----> B -----> C -----> D
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Retiming: Path Weights over all paths

- Paths weights = synchronous delay = # registers
- For each $v_i, v_j$, $W(v_i, v_j) = \min w(v_i, ..., v_j)$ over all paths from $v_i$ to $v_j$. Paths from a to e: 2 paths:
  - a-b-c-d-e: $w = 3$. Whereas, a-b-c-e: $w = 2$.
  - $W(a, e) = \min (2, 3) = 2$. 

![Graph showing path weights](image-url)
Retiming: Propagation Delay over all paths

- Propagation delay of N/W: $d-e-f-g-h = 24$
- For each $v_i, v_j$, $D(v_i, v_j) = \max d(v_i, ..., v_j)$ over all paths from $v_i$ to $v_j$ \textbf{with weight} $W(v_i, v_j)$.

- $a->e$: 2 paths: $a-b-c-d-e + a-b-c-e$: $W(a, e) = 2$.
  - $D(a, e) = \max (16) = 16$. Only 1 2-register path
  - $D$: Max delay on $a-e$ w/ min registers between $a-e$. 

![Diagram of graph with labeled edges and vertices]
Retiming: Timing Feasibility

- Let $t =$ required cycle time. N/W is timing feasible, if for all paths $v_i$ to $v_j$, $D(v_i, v_j) \leq t$.
- IOW, if $D(v_i, v_j) > t$, there should be at least 1 register on the path $\Rightarrow W(v_i, v_j) \geq 1$.
- For all $(v_i, v_j)$, if $D(v_i, v_j) > t$, the $W(v_i, v_j) \geq 1$

![Diagram](image)
Delay Feasible Retimings of a N/W

- Let \( t \) = cycle time. Retiming vector \( \mathbf{r} \) is feasible iff:
  - Retiming consistence constraint: \( w'_{ij} = w_{ij} + r_j - r_i \geq 0 \) (cannot remove more than what you have) for every \( v_i, v_j \)
  - Timing Feasibility constraint: \( W'_{ij} \geq 1 \) for all \( D_{ij} > t \)
  - \( W' = W_{ij} + r_j - r_i \geq 1 \)
Retiming Feasibility Check!

- Let $t = 13$. Retiming vector $r = [-11222100]$
- Let $v_i = a$ and $v_j = e$.
  - Show $(w_{ae} + r_e - r_a) \geq 0$)
- For path $a-e$, $D = 16 > t = 13$. Prove that $a-e$ is broken by at least 1 register
  - $W'(a, e) = W(a, e) + r_e - r_a \geq 1$
Retiming Algorithm for Delay

- Complicated Computations.....
- Compute all path weights between every pair of nodes: \( (w_{ij}) \) between all nodes i and j.
  - Get \( W(i, j) = \min w(i, ..., j) \)
- Similarly, get \( D(i, j) \).
- Construct all inequalities.
- Sort the values of \( D \).
- Start from smallest \( D_i \)
  - Check for feasible retiming
    - Solve inequalities (MILP, or Bellman-Ford)
    - If solved, Determine vector \( r: \) (Retimed for Delay \( D_i \))
- If not, get next \( D \) and continue......
Retiming for Minimum Area – Register Minimization

- Let $v$ be a vertex:
  - In-degree($v$) = 2; Out-degree($v$) = 1
  - Forward retiming: $r(v) = -1$
  - Variation in register count: $r \cdot (\text{in-deg} - \text{out-deg})$
    - $= (-1)(2 - 1) = -1 \Rightarrow$ reduction in one reg
  - Backward retiming: $1 \cdot (2 - 1) = 1$.

- Retime for Area:
  - Minimize (sum of all variations in reg counts at local nodes)
  - Anything else?