Performance of Channel Coded Noncoherent Systems: Modulation Choice, Information Rate, and Markov Chain Monte Carlo Detection

Rong-Rong Chen, Member, IEEE, and Ronghui Peng, Student Member, IEEE

Abstract—This paper investigates performance of channel coded noncoherent systems over block fading channels. We consider an iterative system where an outer channel code is serially concatenated with an inner modulation code amenable to noncoherent detection. We emphasize that, in order to obtain near-capacity performance, the information rates of modulation codes should be close to the channel capacity. For certain modulation codes, a single-input single-output (SISO) system with only one transmit antenna may outperform a dual-input and single-output (DISO) system with two transmit antennas. This is due to the intrinsic information rate loss of these modulation codes compared to the DISO channel capacity. We also propose a novel noncoherent detector based on Markov Chain Monte Carlo (MCMC). Compared to existing detectors, the MCMC detector achieves comparable or superior performance at reduced complexity. The MCMC detector does not require explicit amplitude or phase estimation of the channel fading coefficient, which makes it an attractive candidate for high rate communication employing quadrature amplitude modulation (QAM) and for multiple antenna channels. At transmission rates of 1 ∼ 1.667 bits/sec/Hz, the proposed SISO systems employing 16QAM and MCMC detection perform within 1.6-2.3 dB of the noncoherent channel capacity achieved by optimal input.

Index Terms—Noncoherent detection, Markov Chain Monte Carlo, fading channel, multiple antenna, transmit diversity, iterative decoding, channel capacity.

I. INTRODUCTION

In this paper, we study channel coded noncoherent systems where neither the transmitter nor the receiver has explicit channel information a priori. There has been much work on the design of modulation codes and noncoherent detection algorithms for both single-input single-output (SISO) systems [1], [2] and multiple antenna systems including the dual-input and single-output (DISO) systems [3], [4]. However, only limited research has addressed the effect of information rates of modulation codes and transmit diversity on the performance of noncoherent channel coded systems. For instance, an interesting question is that, is it safe to assume that a system with two transmit antennas, as in a DISO system, automatically performs better than a SISO system with only one transmit antenna? Our results show that adding a second transmit antenna does not always enhance performance.

In this work, we point out an important design criterion for channel coded system. That is, in order to obtain near capacity performance, one should choose modulation codes whose mutual information rates are close to the optimal channel capacity. We provide an explicit comparison of the mutual information rates between a simple 16QAM modulation code for the SISO channel, and the unitary space-time modulation (USTM) code [3] and pilot-symbol assisted modulation (PSAM) code [4] for the DISO channel. It is shown that the mutual information rates of USTM and PSAM codes are much lower than that of the 16QAM code for SISO channel, which implies that they fall well below the DISO channel capacity. This contributes to the fact that such DISO systems perform even worse than the SISO system. To the best of our knowledge, this is the first work to investigate the effect of transmit diversity for channel coded noncoherent systems through an explicit comparison of SISO and DISO systems.

Furthermore, we propose a novel noncoherent detector based on the Markov Chain Monte Carlo (MCMC) method. It differs from existing detectors [1][4] in that it does not require amplitude estimation or phase quantization of the channel fading coefficient. It also differs from the detector of [2] that employs linear prediction and per-survivor processing. We provide detailed performance and complexity comparisons between the proposed detector and the detector of [4]. The latter is shown to obtain near-optimal soft information [5].

Noncoherent MCMC detectors are first studied by X. Wang et al. [6]–[9] for OFDM systems and multicarrier CDMA systems. The noncoherent MCMC detector proposed in this paper originates from coherent MCMC detectors of [10]–[13]. Such MCMC detectors require neither the burning period nor bit-counting for computing a posteriori probabilities [11]. They significantly outperform traditional MIMO detectors such as the sphere decoding detector. Detailed differences between the proposed detector and those of [6]–[9] are highlighted in Section IV.

II. SYSTEM MODEL

We consider a SISO block fading channel where the channel remains constant for each block of $T_c$ symbols (where $T_c$ is called the coherence length), and is independent between blocks. We model the channel by:

$$y = \sqrt{\rho} h s + w,$$  \hspace{1cm} (1)
where $h \sim \mathcal{CN}(0, 1)$ is the Rayleigh fading coefficient of a given block and is a circularly symmetric complex Gaussian random variable with zero mean and unit variance; the vectors $y, s, w$ are $T_c$-dimensional complex vectors representing the received signal, the transmitted signal, and the noise, respectively; the entries of $w$ are independent and identically distributed with distribution $\mathcal{CN}(0, 1)$. The constant $\rho$ represents the signal-to-noise ratio (SNR), assuming that the average power of the transmitted signal $s$ is normalized such that $E|s|^2 = T_c$, where $\dagger$ denotes the Hermitian operator. Given $s$, the noncoherent conditional probability density function (pdf) of $y$ is given by [14]:

$$p(y|s) = \frac{1}{\pi R_c(1 + \rho|s|^2)} \exp\left\{-\frac{|y|^2 + \rho|y^\dagger s|^2}{1 + \rho|s|^2}\right\}$$ (2)

Fig. 1 shows a block diagram of the channel coded noncoherent system. At the transmitter, a simple modulation code is used to map an input block of $(T_c - 1)$ complex symbols to an output block of $T_c$ symbols by inserting a reference symbol $c_0$ in the front of each input block: $(s_1, \ldots, s_{T_c-1}) \rightarrow s = (s_0 = c_0, s_1, \ldots, s_{T_c-1})$, where each $s_i, i = 1, \ldots, T_c-1$ represents $M_c$ bits. The overall transmission rate of this system is $R = \frac{T_c}{T_c-1} R_c M_c$, where $R_c$ is the rate of the channel code. Joint iterative channel decoding and noncoherent detection is performed at the receiver. Detailed description of the block diagram can be found in [15].

III. INFORMATION RATE OF MODULATION CODE AND ITS IMPACT ON CODED PERFORMANCE

From a system perspective, the selection of modulation codes is important because their mutual information rates determine the maximum information rate that a coded system can achieve for a given SNR, denoted by $E_s/N_0$. In other words, for a desired transmission rate $R$, the information rate of the modulation code determines the minimum $E_s/N_0$, denoted by $E_s/N_0|_{\text{min}}$, required to achieve $R$. Note that this is an information-theoretical limit that can be achieved only with optimal detection and a powerful channel code with an arbitrarily long code length and maximum-likelihood decoding. Hence, it is also the performance limit of any practical channel coded system with suboptimal detectors and iterative decoding.

In this section, we examine the information rate of the modulation code defined in Section II for SISO channel and compare with those of certain modulation codes used for DISO channels. This will explain the performance gap between the proposed SISO system and that of the DISO systems in [4], shown in Section V.

![Block diagram of the channel coded noncoherent system](image)

**Fig. 1.** A schematic block diagram of the channel coded noncoherent system.

A. Information rates of practical modulation codes for SISO channel

We first consider a SISO noncoherent block fading channel with $T_c = 6$. Fig. 2 plots the channel capacity achieved by the optimal input and the mutual information rates [1] of the modulation code described in Section II using practical constellations 16QAM, 8QAM, 8PSK, and QPSK (a similar figure for $T_c = 5$ was presented in [1]).

From Fig. 2, we make two observations. First, the mutual information rates of modulation codes provide performance benchmarks for channel coded systems. For instance, Fig. 2 shows that, to achieve $R = 1.667$ using the 16QAM modulation code, we have $E_s/N_0|_{\text{min}} = 8$ dB. This is independent of the choices of detection algorithms and channel codes. Second, among the practical constellations considered here, 16QAM can best approximate the channel capacity because it achieves the highest information rate. For instance, when $R = 1.667$, we have $E_s/N_0|_{\text{min}} = 7.5, 8, 8.7$ dB, respectively, for the optimal input, 16QAM, and 8QAM. Hence, 16QAM is better than 8QAM, because the $E_s/N_0|_{\text{min}}$ required is only 0.5 dB away from that of the optimal input.

B. Comparisons of information rates for SISO system and DISO system

To facilitate low-complexity noncoherent detection, modulation codes such as USTM are often employed in practical DISO systems [3], [4]. Unfortunately, these codes suffer from intrinsic information rate loss compared to the optimal channel capacity. In [16], it is shown that the information rates of USTM achieve only a fraction of channel capacity. In [4], the 512-ary USTM and the 256-ary QPSK/Alamouti modulation codes are considered for a DISO channel. To achieve $R = 1$, these two codes require $E_s/N_0|_{\text{min}} = 8.15$ dB and $8.45$ dB, respectively, which are about 4 dB more than the $E_s/N_0|_{\text{min}} = 4.2$ dB required for a SISO channel with 16QAM (see Fig. 2). For $R = 1.5$, the DISO channel with 8PSK/Alamouti code
requires $E_b/N_0|_{\text{min}} = 11.76$ dB [4], while a SISO channel with 16QAM needs only $E_b/N_0|_{\text{min}} = 7.1$ dB. Simulation results in Section V will verify that, the proposed SISO system with 16QAM indeed outperforms the DISO systems with the modulation codes above by about 4 dB. These comparisons clearly show that the information rates of these modulation codes used for DISO systems are much lower than those of the modulation codes used for SISO systems. Therefore, for such scenarios a SISO system should be chosen over a DISO system, and one should not waste the resource of a second transmit antenna.

We emphasize that, to obtain capacity-approaching performance, it is important to choose modulation codes whose distribution is still unknown. The single antenna performance. However, the capacity of a DISO channel should be no less than that of a SISO channel, because with dual transmit antennas, one can always choose to allocate full power to one of the transmit antennas to realize the single antenna performance. However, the capacity of the DISO channel is achieved only with the optimal input, whose distribution is still unknown.

IV. NONCOHERENT DETECTION BASED ON MARKOV CHAIN MONTE CARLO (MCMC)

In this section, we propose a noncoherent MCMC detector based on the coherent MCMC detectors of [10]–[13] where coherent detection is employed assuming perfectly known channel fading coefficient. Here, we extend the basic idea of MCMC detection to the noncoherent scenario where the channel fading coefficient is unknown. The proposed noncoherent MCMC detector computes extrinsic log-likelihood ratios (LLR) of the coded bits based on received signal vector $y$ and prior LLRs $\{\lambda_i\}$ provided by the channel decoder. Given a modulation codeword $s = (s_0, s_1, \cdots, s_{T_e-1})$, we denote the bit sequence corresponding to $\{s_1, \cdots, s_{T_e-1}\}$ by $b = (b_1, b_2, \cdots, b_K)$, where $K = (T_e-1)M$. In particular, the $M$ bits corresponding symbol $s_i$ are $b_{iM+1}, \cdots, b_{iM}$. Each bit $b_i$ equals either 0 or 1. The MCMC detector operates in two steps described below.

Step 1: Use Gibbs sampler to identify a small set of $I$ “likely bit vectors”, denoted by $\mathcal{A}$.

Initialization $n = 0$; generate the initial vector $b^{(0)} = \{b_1^{(0)}, \cdots, b_K^{(0)}\}$ according to (3).

for $n = 1$ to $I$

for $i = 1$ to $K$

Sample $b_i^{(n)}$, the $i$-th bit of $b^{(n)}$, according to the a posteriori probability distribution $\pi$:

$$b_i^{(n)} \sim \pi(y|c_0, b_1^{(n)}, \cdots, b_{i-1}^{(n)}, b_{i+1}^{(n)}, \cdots, b_K^{(n)}, \lambda_i).$$

end $i$ loop

end $n$ loop

First, the initialization step to find $b^{(0)}$ is done as follows. For each $i = 1, \cdots, T_e - 1$, we compute the most likely transmitted symbol $\hat{s}_i$ based on the received signals $y_0$ and $y_i$ by letting

$$\hat{s}_i = \arg\max_{s_i} \{ \ln p(y_0, y_i|c_0, s_i) + \ln P(s_i) \},$$

where $P(s_i) = \sum_{j=(i-1)M+1}^{iM} (\lambda_j/2)(-1)^b_j$ is the logarithm of the prior probability of symbol $s_i$, and $\lambda_j$ is the prior LLR of the $j$-th bit. The pdf $p(y_0, y_i|c_0, s_i)$ in (3) is the noncoherent pdf corresponding to $T_e = 2$ because only two signals $y_0$ and $y_i$ are considered. The symbol $\hat{s}_i$ is then used to define the initial bit vector $b^{(0)}$ by letting $(b_1^{(0)}, b_2^{(0)}, \cdots, b_K^{(0)})$ equal to the bits constituting symbol $\hat{s}_i$.

In the step of sampling $b_i^{(n)}$, we let

$$x = \ln \frac{p(y|c_0, b_0^{(n)}, \cdots, b_{i-1}^{(n)}, b_{i+1}^{(n)}, \cdots, b_K^{(n)})}{p(y|c_0, b_0^{(n)}, \cdots, b_{i-1}^{(n)}, 1, b_{i+1}^{(n)}, \cdots, b_K^{(n)})} + \lambda_i,$n

$$and$$t = e^x/(1 + e^x),$$

where the pdf in (4) is computed using (2). We then generate a random number $u \in [0, 1]$ according to the uniform distribution. If $u < t$, we let $b_i^{(n)} = 0$, otherwise we let $b_i^{(n)} = 1$.

Step 2: Compute the output extrinsic LLR $\{\gamma_i\}$ based on vectors in $\mathcal{A}$.

For each bit vector $b \in \mathcal{A}$, by replacing its $i$-th bit by 0 and 1, respectively, and leaving other bits unchanged, we obtain two new bit vectors $b^{i,0}$ and $b^{i,1}$. These vectors are used to compute the output extrinsic LLR $\gamma_i$ for bit $i$:

$$\gamma_i = \max_{b \in \mathcal{A}} [\ln p(y|c_0, b^{i,0}) + \ln P(b^{i,0})] - \max_{b \in \mathcal{A}} [\ln p(y|c_0, b^{i,1}) + \ln P(b^{i,1})] - \lambda_i,$n

where $P(b^{i,0}) = \sum_{j=1}^{K} (\lambda_j/2)(-1)^b_j$ and $b_j^{i,0}$ denotes the $j$-th bit of $b^{i,0}$. The term $\ln P(b^{i,1})$ is computed similarly.

The proposed MCMC detector differs from the MCMC detectors of [6]–[9] in both initialization (3) and computation of output LLRs (5). In [6]–[9], bit-counting (use statistical averaging to estimate the frequency that a particular bit value occurs) is applied to compute the LLRs. In comparison, we use (5) to compute the LLRs based on the a posteriori probabilities of the samples generated by the Gibbs sampler. The proposed detector does not require a burning period and only a small number of samples are needed to achieve satisfactory performance. Detailed analysis of the proposed noncoherent MCMC detector resembles those of [11] for coherent MIMO channels. Since the main complexity of the MCMC detector comes from the computation of the noncoherent pdf (CNP) in (4) and (5), the total number of CNP represents the complexity of the MCMC detector fairly accurately. The proposed MCMC detector requires approximately $2IK$ CNPs [15].

V. SIMULATION RESULTS

As described in Section IV, the MCMC detector has the unique feature that it does not require any phase or amplitude quantization of channel fading coefficient. In this section, we compare the MCMC detector with (1) the bit-flipping (BF)
detector of [4] that uses phase quantization, and (2) a genie-aided bit-flipping detector (G-BF) which differs from the BF detector only in that it assumes perfect knowledge of the channel fading amplitude. The BF detector is shown to be near optimal for channels with small or moderate coherence lengths [5]. In terms of CNPs, its complexity equals $QK$, where $Q$ is the number of phase quantization, and $K = (T_c - 1)MC_c$ is the total number of bits transmitted in each fading block. Section IV shows that the MCMC detector has a complexity of $2JK$ CNPs. To facilitate fair comparisons, we consider channel coded systems using the same channel code, which is a commonly used regular $(3, 6)$ low-density parity-check (LDPC) code with rate $R_c = 1/2$ and code length $10^4$, and the same 16QAM modulation code described in Section III. The overall rate of the system, hence, equals $R = \frac{T_c - 1}{4}R_c$, corresponding to $R = 1.667$ for $T_c = 6$ and $R = 1.933$ for $T_c = 30$, respectively. Fig. 3 shows the bit-error-rate (BER) of the coded system versus the average energy per information bit to noise ratio $E_b/N_0$. It relates to $E_b/N_0$ by $\frac{Eb}{N_0}\text{dB} = \frac{E_b}{N_0}\text{dB} - 10\log_{10} R$. Detailed simulation parameters can be found in [15].

Fig. 3 shows that for a fast fading scenario $T_c = 6$ ($R = 1.667$), MCMC detector with $I = 3$ performs about 0.05 dB better than BF detector with $Q = 6$ at BER $= 10^{-4}$. The complexity of these two detectors are roughly the same in terms of CNPs. When $Q = 10$, at the cost of higher complexity, performance of BF detector improves slightly and is virtually the same as MCMC detector. At BER $= 10^{-4}$, G-BF detectors with $Q = 10$ and $Q = 6$, assuming perfect fading amplitude, achieve about 0.08 dB and 0.05 dB gain over the MCMC detector that assumes unknown fading amplitude. Note that if BF detector is modified to perform amplitude quantization in addition to phase quantization, its performance will be inferior than that of the G-BF detector despite higher receiver complexity. For a slow fading scenario $T_c = 30$ ($R = 1.933$), with roughly the same complexity, MCMC detector outperforms BF detector with $Q = 6$ by about 0.44 dB at BER $= 10^{-4}$. Even with an increased complexity, the BF detector with $Q = 10$ still performs about 0.19 dB worse than the MCMC detector. The G-BF detector with $Q = 6$ and $Q = 10$, assuming perfect amplitude, still perform worse than the MCMC detector that assumes unknown amplitude by about 0.28 dB and 0.08 dB, respectively.

To further approach channel capacity, we optimize the LDPC code following the extrinsic information transfer (EXIT) chart approach [17]. The optimized code parameters are given in [15]. Performance of the optimized system using MCMC detection is shown in Fig. 4. For $R = 1.667$, with 16QAM and an optimized LDPC code rate $R_c = 1/2$, the channel coded system achieves within 1.8 dB of the capacity limit of 16QAM ($\frac{Eb}{N_0}\text{dB} = 5.78$ dB) at BER $= 10^{-4}$, and is 2.3 dB away from the capacity limit of the optimal input. For $R = 1$, with 16QAM and an optimized code rate $R_c = 0.3$, we achieve within 1.2 dB of the capacity limit of 16QAM ($\frac{Eb}{N_0}\text{dB} = 4.2$ dB), and is 1.6 dB away from the capacity of the optimal input. Compared to DISO systems [4] at $R = 1$ and $R = 1.5$, the proposed SISO system achieves about 4 dB performance gain. This is consistent with our observation in Section III that, at these transmission rates, the $\frac{Eb}{N_0}\text{dB}$ required by the 16QAM code is about 4 dB less than that of the modulation codes used in [4]. While Fig. 2 shows that $\frac{Eb}{N_0}\text{dB}$ for 16QAM and 8QAM differ by only 0.2 dB at $R = 1$, we see from Fig. 4 that the 8QAM system performs about 0.6 dB worse than the 16QAM system due to the use of a higher rate channel code with $R_c = 0.4$.

VI. CONCLUSION

This paper studies performance of noncoherent channel coded systems. We show that transmit diversity does not necessarily enhance performance when there is a large gap
between the mutual information rates of modulation codes and the optimal channel capacity. This explains the interesting but somewhat surprising fact that the proposed SISO systems significantly outperform certain DISO systems by as much as 4 dB. While in this work we focus on systems with single receive antenna, the basic principles presented are applicable to general scenarios with multiple receive antennas. For instance, our preliminary results show that even with dual receive antennas, systems with one transmit antenna can still outperform dual transmit antenna systems employing similar modulation codes discussed here for the DISO channel. An interesting direction for future work is to design better modulation codes for multiple transmit antenna channels that can fully exploit the channel capacity and also allow for low-complexity detection. Excellent performance of the proposed MCMC detector for the SISO channel demonstrates that it will be instrumental in the design of capacity-approaching noncoherent systems for multiple antenna channels.

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