3. a) Calculate the value of $R_L$ that would absorb maximum power.

Solv.: Transform into Thevenin equivalent of circuit left of $R_L$.

Use superposition to find $V_{Th}$:

\[ V_{Th} = V_1 + V_2 \]

In both cases, we have no current thru $100\Omega$ top right. So no $V$-drop across it.

\[ v_1 \text{ from } V\text{-divider: } \quad v_2 = 2A \cdot 100\Omega || 100\Omega \]

\[ v_1 = 100V \cdot \frac{100\Omega}{100\Omega + 100\Omega} = 50V \quad v_2 = 2A \cdot \frac{50\Omega}{100\Omega} = 100V \]

\[ V_{Th} = v_1 + v_2 = 150V \]

For $R_{Th}$, we turn 1 sources to zero and see what resistance we have looking in from a-b terminals:

\[ R_{Th} = 100\Omega || 100\Omega = 150\Omega \]

Max power xfer when \[ R_L = R_{Th} = 150\Omega \]
3. b) Calculate the value of max power, $R_L$ chosen for max power.

**sol'n:** Our circuit with $R_L = R_{th}$ is:

\[ 150V \quad + \quad + \quad \frac{150\Omega}{N} \quad \frac{i}{150\Omega} \quad + \quad 150\Omega \quad v \quad \text{pwr} \quad p = i \cdot v = \frac{150V \cdot 75V}{500\Omega} \]

\[ p = 37.5 \text{ W} \]

Use superposition. Find expression for $v_3$ not using $i_x$.

**sol'n:** Turn one independent source on at a time, with other independent sources $= 0$. Sum currents and $v$'s from each of the models to get total currents and voltages for circuit.

Note: Current source set to 0 is open circuit.
Voltage $\rightarrow$ short circuit, (i.e. wire).

Circuit 1: $i_s \neq 0, \quad v_f = 0$. Add a "f" subscript to $i_s$ and $v_f$.

\[ i_{x1} = \frac{V_{31}}{R_1} \]

Use Node V for $V_{31}$:

\[ i_s = i_{x1} + \frac{V_{31} - \beta i_{x1}}{R_2} = \frac{V_{31} + V_{31} - \beta V_{31}}{R_1 + \frac{1}{R_2} - \frac{\beta}{R_1 R_2}} \]

\[ i_s = V_{31} \left( \frac{1}{R_1} + \frac{1}{R_2} - \frac{\beta}{R_1 R_2} \right) \text{ or } V_{31} = i_s \left( \frac{1}{R_1} + \frac{1}{R_2} - \frac{\beta}{R_1 R_2} \right) \]
4. (cont) Circuit 2: 

\[
V_S = 0, \quad i_S = 0. \quad \text{Add a "2" subscript to } i_S \text{ and } V_S.
\]

Use sum \( V_S \) around loop = 0. (KVL)

\[
V_S + i_{x2} = \frac{V_S}{R_1} + \frac{V_S}{R_2}
\]

or \( V_S = \frac{V_{32} R_2 + \beta V_{32} - V_{32}}{R_1} = 0 \)

\[
V_3 = V_{32} \left( \frac{R_2 + \frac{1}{\beta}}{R_1} \right)
\]

\[
V_{32} = \frac{V_S}{R_1 + \frac{1}{\beta}}
\]

Now \( V_3 = V_{31} + V_{32} \),

\[
eqn 1 \quad v_3 = \left( \frac{i_S + V_S}{R_2} \right) \left( \frac{1 + \frac{1}{\beta}}{R_1} \right)
\]

\[
eqn 2 \quad v_3 = \left( \frac{i_S + V_S}{R_2} \right) \frac{R_1 + R_2}{\beta}
\]

Note: \( R_a || -R_b \equiv \frac{R_a R_b}{R_a + R_b} \)

\[
eqn 3 \quad v_3 = \left( \frac{i_S R_2 + V_S}{R_1} \right) \left( \frac{R_2 + \frac{1}{\beta}}{R_1} \right) = \frac{R_a (-R_b)}{R_a + (-R_b)}
\]

Consistency Checks:
1) \( \beta = 0, \quad v_3 = 0 \rightarrow v_3 = i_S \left( R_1 || R_2 \right) \)

2) \( R_1 = \infty \rightarrow v_3 = V_S + i_S R_2 \)

3) \( i_S = 0, \beta = 0 \rightarrow v_3 = V_S \frac{R_1}{R_1 + R_2} \)

4) \( R_1 = 1 \Omega, \quad R_2 = 2 \Omega, \quad V_S = 3V, \quad i_S = 4A, \quad v_3 = 4V, \quad \beta = 8 \)

\[
\begin{align*}
\beta i_S &= 4V, \\
V_{R_2} &= i_S (V_S + \beta i_S) = 4 \frac{1}{2} = 2V, \\
i_S &= \frac{i_S}{R_2} = \frac{4}{2} = 2A, \\
V_S &= (4 + 3) = 7V \\
\end{align*}
\]

\[
eqn 2 \quad v_3 = \left( \frac{0 + V_S}{R_2} \right) \left( \frac{R_1 || R_2}{\beta} \right) = V_S \frac{R_1}{R_1 + R_2} \]

\[
eqn 2 \quad v_3 = \left( \frac{i_S + V_S}{R_2} \right) \frac{R_1 + R_2}{\beta} = V_S \frac{R_1 R_2}{R_1 + R_2}
\]

\[
eqn 2 \quad v_3 = \left( \frac{i_S + V_S}{R_2} \right) \frac{R_1 + R_2}{\beta} = V_S \frac{R_1 R_2}{R_1 + R_2}
\]