Given an ideal transformer in the above circuit, calculate \( i(t) \).

and: \[ i(t) = 6 \cos \left( \frac{20 \cdot 10^4 t + 143^\circ}{9} \right) \text{A} \]

**Sol'n:** As shown in the Text, we have the following relationships for currents and voltages:

\[
\frac{V_1}{V_2} = \frac{N_1}{N_2} \quad \frac{I_1}{I_2} = \frac{N_2}{N_1}
\]

or \[ V_2 = \frac{N_2}{N_1} V_1 \] or \[ I_2 = \frac{N_1}{N_2} I_1 \]

Note: We have positive signs in these eqns if we define \( V_1, V_2, I_1, \) and \( I_2 \) as shown in the diagram above. In particular \( I_1 \) is in the direction of the passive sign convention, but \( I_2 \) is opposite the passive sign convention. Also, \( V_1 \) and \( V_2 \) are measured with the plus sign on the dotted side of transformer windings.

For the ideal transformer, we do not know what the impedances inside the transformer are. Thus, we must use only the ideal transformer relationships to write eqns from which we solve for \( I_1 \) (or \( i(t) \)).

We write mesh current eqns for the three loops and then use the ideal transformer relationships.
Mesh loop eq's from top to bottom:

\[-I_2 \cdot R_z - V_2 = 0 \quad \text{or} \quad I_2 R_z + V_2 = 0 \quad (1)\]

\[V_2 - V_1 - \frac{1}{j\omega C} (I + I_2) = 0 \quad (2)\]

\[V_1 - R_1 (I + I_2 - I_1) - V_3 = 0 \quad (3)\]

Use ideal transformer relationships to eliminate \(I_2\) and \(V_2\):

\[I_2 = \frac{N_1}{N_2} I_1 = \frac{I_1}{10}\]

Also, \[\frac{1}{j\omega C} = -j9\]

\[V_2 = \frac{N_2}{N_1} V_1 = 10V_1\]

\[I_2 R_z + 10V_1 = 0 \quad \text{or} \quad V_1 = -\frac{R_z}{10} I = -I\quad (1)\]

\[10V_1 - V_1 + j9 (\frac{I + I_1}{10}) = 0 \quad (2)\]

\[V_1 - R_1 (\frac{I + I_1 - I_1}{10}) - V_3 = 0 \quad (3)\]

Now use (1) to eliminate \(V_1\) in (2) and (3):

\[(-10 \quad + 
\quad I \quad + 
\quad j9 \quad I \quad + 
\quad j9 \quad I_1) = 0 \quad (2)\]

\[-I - R_1 (I - j9I_1) - V_3 = 0 \quad (3)\]

Now use (2) to express \(I_1\) in terms of \(I\):

\[I_1 = \frac{j10}{9} (-9 + j9) I = j10(-1+j) I\]
Substitute for $I_t$ in (3):

\[- I - \frac{1}{2} \left( I - \frac{q}{10} j \alpha (-1 + j) I \right) - V_s = 0\]

\[\left[ - \frac{3}{2} + \frac{1}{2} j (-1 + j) \right] I - V_s = 0\]

\[\left(-\frac{3}{2} - \frac{1}{2} j \frac{1}{2}\right) I - V_s = 0\]

\[\left(-\frac{1}{2} - j \frac{1}{2}\right) I - V_s = 0\]

\[\mathbf{I} = \frac{V_s}{-6 - j \frac{1}{2}} = -\frac{45}{6 + j \frac{1}{2}} = -\frac{15}{2 + j \frac{3}{2}}\]

\[\mathbf{t} = -\frac{30}{4 + j \frac{3}{2}} = -\frac{30}{(4 - j^3)} = -\frac{6}{5} \left(4 - j^3\right)\]

\[\mathbf{t} = -\frac{6}{5} \cdot 5 \angle -37^\circ\]

\[\mathbf{t} = \frac{6}{5} \cdot 5 \angle 180^\circ - 37^\circ\]

\[\mathbf{I} = 6 \angle 143^\circ \text{ A}\]

\[\therefore \mathbf{i(t)} = 6 \cos (\omega t + 143^\circ) \text{ A} \quad \omega = \frac{20 \cdot 10^4 \text{ rad/s}}{9}\]