The above circuit is from Lab 4, but a number of circuit changes have been made. Note that voltage $v_2$ is a triangle waveform, and $v_3$ is a Pulse-Width Modulation (PWM) waveform.

Plot $v_2$ and $v_3$. Assume $v_2 = 0V$ at $t=0$.

a) We start by finding the value of $v_2$ where the first comparator op-amp switches.

Suppose $v_1$ is high. $v_1 = 15V$ supply - $1V = 14V$. That is, $v_1 = 14V$, which is the positive rail voltage.

$v_1$ will switch when the voltages at the + and - inputs are equal. That is, $v_1$ switches when $v_+ = v_- = 0V$.

We have the following picture:

We have $v_+ = \frac{v_1 R_1 + v_2 R_2}{R_1 + R_2}$

Now we solve for $v_2$.  

\[ v_+ = \frac{v_1 R_1}{R_1 + R_2} + \frac{v_2 R_2}{R_1 + R_2} \]

or
\[ OV = \frac{v_1 R_1}{R_1 + R_2} + \frac{v_2 R_2}{R_1 + R_2} \]

or
\[ \Delta v = v_1 R_1 + v_2 R_2 \]

or
\[ v_2 = -\frac{v_1 R_1}{R_2} = -14V \cdot \frac{10k\Omega}{14k\Omega} = -10V \]

By symmetry, \( v_2 \) will switch at \( \pm 10V \).

\[ v_2 \uparrow 10V \]
\[ -10V \downarrow \]
\[ t/4 \text{ (one-fourth of period)} \]

We now find the slope of \( v_2 \).

When \( v_1 \) is high, we have current \( i = \frac{v_1}{R_3} \)

because \( v_- = v_+ = 0V \).

Current \( i \) changes \( C \) : \[ v_c(t) = \frac{1}{C} \int_0^t i(t)\,dt + v_c(0) \]

Using \( v_c(0) = 0V \), since \( v_2(0) = 0 \), we have

\[ v_c(t/4) = 10V = \frac{1}{C} \int_0^t \frac{v_1}{R_3} \,dt = \frac{1}{R_3C} \int_0^t \frac{14V \pm}{14k\Omega \cdot \frac{1}{2} \mu F} \]

or

\[ v_c(t/4) = 10V = 2kV/s \cdot t \Rightarrow t = 5 \text{ ms} \]
b) $v_3$ is the output of a comparator. $v_3$ will be equal to $+v_{rail} = 15V - 1V = 14V$ when $v_+ > v_-$. That is, $v_+ > v_2$.

We have a $v$-divider for $v_+$:

$$v_+ = 15V \cdot \frac{3k\Omega}{3k\Omega + 12k\Omega} = 3V$$

So the comparator output is high when $v_2 < 3V$.

The falling edge of $v_3$ occurs when $v_2 = 3V$.

$$v_2 = \frac{10V \cdot t}{5ms} = 3V \Rightarrow t = \frac{3V(15ms)}{10V} = 1.5ms$$