1. For a cascade MOS mirror utilizing devices with \(V_t=0.5\text{V}, \mu_nC_{\text{ox}}=387\text{µA/V}^2, W/L=3.6\text{µm}/0.36\text{µm},\) \(V_A'=10\text{V/µm},\) and \(I_{\text{REF}}=100\text{µA}.\) Find the minimum voltage required at the output and the output resistance.

\[
I_{D1} = I_{D2} = I_{D3} = I_{D4} = I_{\text{REF}} = 100\, \text{µA}
\]

Since \(I_{D} = \frac{1}{2} \mu_n C_{\text{ox}} \left( \frac{W}{L} \right) V_{\text{OV}}^2\),

\[
V_{\text{OV}} = \frac{2I_{D}}{\mu_n C_{\text{ox}} \left( \frac{W}{L} \right)} = \frac{2(100\, \text{µA})}{(387\, \text{µA/V}^2) \left( \frac{3.6}{0.36} \right)} = 0.23\, \text{V}
\]

The minimum output voltage is

\[
V_{\text{OV}} + 2V_{\text{BV}} = 0.5\, \text{V} + 2(0.23\, \text{V}) = 0.96\, \text{V}
\]

To obtain the output resistance, \(R_O\), we need \(g_{m3}\).

\[
g_{m3} = \frac{I_{D3}}{V_{\text{OV}}/2} = \frac{2(0.1\, \text{µA})}{0.23\, \text{V}} = 0.87\, \text{mA/V}
\]

\[
r_{\text{os}} = r_{\text{os}} = \frac{V_A(L)}{I_{D}} = \frac{(5\, \text{V/µm})(0.36\, \text{µm})}{0.1\, \text{mA}} = 18\, \text{kΩ}.
\]

From eq. (7.77)

\[
R_O = g_m r_{\text{os}} r_{\text{os}} = (0.87\, \text{mA/V})(18\, \text{kΩ})^2 = 282\, \text{kΩ}
\]

2. The BJT in the circuit below has \(v_{\text{BE}}=0.7\, \text{V}, \beta=100\) and \(V_A=50\, \text{V}.\) Find \(R_O.\)

\[
I_E = \frac{V_E - V_{EE}}{R_E} = \frac{-0.7 - (-5)}{10\, \text{kΩ}} = 0.43\, \text{mA}
\]

\[
g_m = \frac{I_c}{V_c} = \frac{0.43\, \text{mA}}{25\, \text{mV}} = 17.2\, \text{mA/V}
\]

\[
r_o = \frac{V_A}{I_c} = \frac{50\, \text{V}}{0.43\, \text{mA}} = 116.3\, \text{kΩ}
\]

\[
r_{\pi} = \frac{\beta}{g_m} = \frac{100}{17.2\, \text{mA/V}} = 5.8\, \text{kΩ}
\]

\[
R_O = r_o [1 + g_m (R_E || I_{\pi})]
\]

\[
R_O = (116.3\, \text{kΩ}) [(116.3\, \text{kΩ}) \parallel (10\, \text{kΩ} || 5.8\, \text{kΩ})] = 7.46\, \text{MΩ}
\]
3. Design the basic BJT differential amplifier shown below to provide a differential input resistance of at least 10kΩ and a differential voltage gain of 100 V/V. The transistor $\beta$ is specified to be at least 100.

The available positive power supply is 5V.

\[ g_m = \frac{I_C}{V_T} \]

\[ I_C = g_m V_T = 20 \text{ mA/V} \cdot 25 \text{ mV} = 0.5 \text{ mA} \]

\[ 1 - 2I_C = 1 \text{ mA} \]

Eqn 8.93 $A_d = \frac{g_m R_C}{R_{id}}$

\[ R_C = \frac{A_d}{g_m} = \frac{100}{20 \text{ mA/V}} = 5 \text{ kΩ} \]

\[ R_{id} \geq 10 \text{ kΩ} \]

\[ A_d = 100 \text{ V/V} \]

\[ V_{CC} = 5 \text{ V} \]

\[ R_{id} = 10^4 = 2r\pi = 2 \times \frac{\beta}{g_m} \]

\[ g_m = \frac{2\beta}{R_{id}} = 20 \text{ mA/V} \]
4. A BJT differential amplifier is biased from a 1mA constant-current source and includes a 200Ω resistor in each emitter. The collectors are connect to \( V_{cc} \) via 12kΩ resistors. A differential input signal of 0.1V is applied between the two bases.

(a) Find the signal current in the emitters \( (i_o) \) and the signal voltage \( V_{be} \) for each BJT. 
(b) What is the total emitter current in each BJT? 
(c) What is the signal voltage at each collector? Assume \( \alpha = 1 \). 
(d) What is the voltage gain realized when the output is taken between the two collectors?

\[
I = 1\text{mA} \quad R_E = 200 \text{V} \quad R_C = 12 \text{kV} \\
V_{id} = 100 \text{mV} \\
r_e = \frac{V_T}{I_e} = \frac{25 \text{ mV}}{1 \text{ mA}/2} = 50 \text{ \Omega} \\
(a) I_r = \frac{V_{id}}{2(r_e + R_E)} = \frac{100 \text{ mV}}{2(250 \text{ \Omega})} = 0.2 \text{ mA} \\
V_{be} = \frac{r_e}{r_e + R_E} \left( \frac{V_{id}}{2} \right) = 10 \text{ mV} \\
(b) \\
i_{E1} = \frac{I_r}{2} + i_r = 0.5 \text{ mA} + 0.2 \text{ mA} = 0.7 \text{ mA} \\
i_{E2} = \frac{1}{2} - i_r = 0.5 \text{ mA} - 0.2 \text{ mA} = 0.3 \text{ mA} \\
(c) \\
V_{C1} = \approx -i_r R_C = -0.2 \text{ mA} \cdot 12 \text{ k\Omega} \\
= -2.4 \text{ V} \\
V_{C2} = +i_r R_C = 0.2 \text{ mA} \cdot 12 \text{ k\Omega} = +2.4 \text{ V} \\
(d) A_d = V_{od}/V_{id} = \frac{4.8 \text{ V}}{100 \text{ mV}} = 48 \text{ V/V} \\

5. Find the voltage gain and the input resistance of the amplifier shown below assuming \( \beta = 100 \).

\[
\frac{v_2}{v_1} = \frac{\alpha R_{C2}}{2r_e + 2R_e} \\
r_e = \frac{V_T}{I_e} \frac{1}{2} = \frac{25 \text{ mV}}{0.4 \text{ mA}/2} = 125 \text{ \Omega} \\
Assuming \alpha \approx 1, \\
\frac{v_2}{v_1} \approx \frac{25 \text{ k\Omega}}{2(125) + 2(125)} = 50 \text{ V/V} \\
R_e = (\beta + 1)(2r_e + 2R_e) \\
= (100)(2(125) + 2(125)) = 50.5 \text{ k\Omega} \\
\]
6. A bipolar differential amplifier with $I=0.5\text{mA}$ utilizes transistors for which $V_A=10\text{V}$ and $\beta=100$ and $R_C=10k\Omega$.

(a) the differential gain
(b) the common mode gain and the CMRR if the bias current I is generated using a simple current mirror.
(c) the common mode gain and the CMRR if the bias current I is generated using a Wilson mirror.

Equivalent

\[ R_{EE} = r_{e3} = \frac{V_A}{I} = \frac{10 \text{V}}{0.5 \text{mA}} = 20 \text{k}\Omega \]

\[ r_{e2} = r_{e1} = r_e = \frac{V_T}{I/2} = \frac{25 \text{mV}}{0.5 \text{mA}/2} = 100 \text{\Omega} \]

Since $\alpha = \frac{\beta}{\beta + 1} = \frac{100}{101} = 1$,

\[ A_d \approx \frac{R_C}{r_e} = \frac{10 \text{k}}{0.1 \text{k}} = 100 \text{\ V/V} \]

(b) $A_{cm} = \frac{\alpha \Delta R_C}{2R_{EE} + r_e} = \frac{0.02)(10 \text{k})}{2(20 \text{\ k}) + 0.1 \text{\ k}}$

\[ = 0.00499 \text{\ V/V} \]

\[ \text{CMRR(dB)} = 20 \log_{10} \left| \frac{A_d}{A_{cm}} \right| = 20 \log_{10} \left| \frac{100}{0.00499} \right| \]

\[ = 86 \text{\ dB} \]

\[ R_c \approx \frac{1}{2} \beta \alpha r_{e3} \]

\[ R_c \approx \frac{1}{2} (100)(20 \text{\ k}) = 1 \text{\ M}\Omega \]

\[ A_{cm} \approx \frac{\Delta R_C}{2R_o + r_o} \approx \frac{(0.02)(10 \text{\ k})}{2(1 \text{\ m}) + 0.1 \text{\ k}} = 0.0001 \text{\ V/V} \]

\[ \text{CMRR(dB)} = 20 \log_{10} \left| \frac{A_d}{A_{cm}} \right| = 20 \log_{10} \left| \frac{100}{0.0001} \right| \]

\[ = 120 \text{\ dB} \]
7. Design a BJT differential amplifier that provides two single-ended outputs (at the collectors). The amplifier is to have a differential gain (to each of the two outputs) of at least 100 V/V, a differential input resistance \( \geq 10 \, \text{k}\Omega \) and a common mode gain (to each of the two outputs) no greater than 0.1 V/V. Use a 2mA current source for biasing. Give the complete circuit with component values and suitable power supplies that allow for \( \pm 2 \text{V} \) swing at each collector. Specify the minimum value that the output resistance of the bias current source must have. The BJTs available have \( \beta \geq 100 \). What is the value of the input common mode resistance when the bias source has the lowest acceptable resistance?

At \( I_c = 1 \text{mA}, \; r_e = 25 \, \Omega \)

\[ R_d = (\beta + 1) \frac{2r_e}{2} = 5.05 \, \text{k}\Omega < 10 \, \text{K} \]

\[ \Rightarrow \text{need emitter resistors} \]

In this case:

\[ R_{d} = (\beta + 1) \left( 2r_e + 2R_e \right) = 10 \, \text{k}\Omega \]

\[ A_d = \frac{R_C}{R_E + r_e} = \frac{100}{2(2 + 1)} \]

\[ \Rightarrow R_C = 10 \, \text{K} \]

\[ A_{cm} = 0.1 \geq \frac{R_C}{2R_o + R_E + r_e} \]

\[ \Rightarrow R_o \geq 50 \, \text{k}\Omega \]

For \( \pm 2 \text{V} \) swing \( V_{cc} = V_{cc} \)

\[ = V_{cc} - \frac{1}{2}R_C = 2 \]

\[ \Rightarrow V_{cc} = 2 + 10^{-3} \times 10^4 = 12 \, \text{V} \]

Choose \( V_{cc} = \pm 15 \, \text{V} \) although 12 V is ok.

\[ 2R_{icm} = (\beta + 1) \left( 2R_o + R_E + r_e \right) \]

\[ \Rightarrow R_{icm} = 5 \, \text{M}\Omega \]