Problem 1.8:
(a) Consider two windings with zero resistance such that
\[
\begin{align*}
L_1 \frac{di_1}{dt} + M_1 \frac{di_2}{dt} &= v_1 \\
L_2 \frac{di_2}{dt} + M_2 \frac{di_1}{dt} &= v_2 
\end{align*}
\]
(1)
The objective of the problem is to show that $M_1$ must be equal to $M_2$ (i.e., that the inductance matrix must be symmetric) for the stored energy to be well-defined. Towards this goal, assume that both currents start from 0 at $t = 0$ and reach a value $I$ for $t = 2T$. Assume that the current $i_1$ rises from 0 to $I$ in $T$ seconds, then stays constant for the next $T$ seconds. On the other hand, the current $i_2 = 0$ for $T$ seconds, then rises from 0 to $I$ between $T$ and $2T$. Compute the energy absorbed by the two windings by integrating the total electrical power $P_{abs} = v_1 i_1 + v_2 i_2$ from 0 to $2T$. Compare the result to the result obtained if the current profiles for $i_1$ and $i_2$ are swapped. The energy must be the same both ways, or else it would be possible to use one profile to go from 0 to $I$ and the other profile to return to zero. In the process, a positive energy could be delivered to the source by a passive system.

(b) The energy computed in part (a) should be of the form
\[
E_{\text{stored}} = \frac{1}{2} (i_1 \quad i_2) \begin{pmatrix} L_1 & M \\ M & L_2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}
\]
(2)
If the energy was negative for some currents $i_1$ and $i_2$, it would be possible to apply currents to the windings and extract energy from a passive system. Therefore, the objective is to find the conditions that $L_1$, $L_2$, and $M$ must satisfy such that $E_{\text{stored}} > 0$ for all $i_1^2 + i_2^2 \neq 0$. In mathematical terms, the inductance matrix is then called positive definite. The result can be obtained by considering two cases: $i_1 = 0$ and $i_1 \neq 0$. A first condition is obtained from the case $i_1 = 0$. For $i_1 \neq 0$, one can write
\[
E_{\text{stored}} = \frac{1}{2} i_2^2 f(x) \quad \text{with} \quad x = \frac{i_2}{i_1}
\]
(3)
and then, find conditions such that $f(x) > 0$ for all $x$. Is it possible to have $M < 0$?