6.1-3 (a) The total antenna pattern is the product of the element/pattern factor and the array factor

\[ F_a(\theta) = f_{\lambda/2}(\theta) g_{\lambda/2}(\theta) AF(\theta) \]

where the element/pattern factor is that of the \(z\)-oriented half-wave dipole, and the array factor is given by

\[ AF(\theta) = \sum_{n=1}^{2} I_n e^{j\beta r'_n} \]

Placing the origin between the two half-wave dipoles

\[ r'_1 = -\frac{\lambda}{4} \hat{z} \quad r'_2 = \frac{\lambda}{4} \hat{z} \]

Since the transmission lines are of equal length and we assume the summer is an equal summer

\[ I_1 = I_2 = 1 \]

and, as always, the unit radial vector is given by equation (C-4)

\[ \hat{r} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} \]

The relevant dot products are

\[ \beta \hat{r} \cdot r'_1 = \beta \left( -\frac{\lambda}{4} \hat{z} \right) \left( \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} \right) = -\beta \frac{\lambda}{4} \cos \theta = -\frac{2\pi \lambda}{4} \cos \theta = -\frac{\pi}{2} \cos \theta \]

\[ \beta \hat{r} \cdot r'_2 = \beta \left( \frac{\lambda}{4} \hat{z} \right) \left( \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z} \right) = \beta \frac{\lambda}{4} \cos \theta = \frac{2\pi \lambda}{4} \cos \theta = \frac{\pi}{2} \cos \theta \]

The array factor is then

\[ AF = e^{-\frac{\pi}{2} \cos \theta} + e^{\frac{\pi}{2} \cos \theta} = 2 \cos \left( \frac{\pi}{2} \cos \theta \right) \]

The element/pattern factor is given by equation (6-7)

\[ f_{\lambda/2}(\theta) g_{\lambda/2}(\theta) = \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \]

Dropping the factor of 2 for correct normalization, the total antenna pattern is

\[ F_a(\theta) = \frac{\cos^2 \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} \]
(b) The antenna pattern for the full-wave dipole is given by equation (6-8)

\[ F_b(\theta) = \frac{\cos(\pi \cos \theta) + 1}{2 \sin \theta} \]

(c) Plot the two current distributions

which are identical. If the current distributions are identical, the antenna patterns must be identical. If we apply the trigonometric identity

\[ \cos^2(\theta) = \frac{\cos 2\theta + 1}{2} \]

to the antenna pattern in part (a) we get the pattern from part (b)

\[ F_a(\theta) = \frac{\cos^2\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} = \frac{\cos(\pi \cos \theta) + 1}{2 \sin \theta} = F_b(\theta) \]
6.1-8 To use the resonant length correction Table 6-2, we first need the aspect ratio of the wire

\[ f = 177 \times 10^6 \text{ Hz} \]
\[ c = 2.9979 \times 10^8 \text{ m/s} \]
\[ a = 0.25 \text{ in} = 0.0635 \text{ m} \]
\[ \lambda = \frac{c}{f} = 1.694 \text{ m} \]

\[ \frac{L}{2a} = \frac{\lambda}{4a} = 66.7 \]

This is pretty close to 50. You could interpolate the data in the table (as I have), but it doesn't make much difference.

\[ L = 0.4751\lambda \approx 80.46 \text{ cm} \]

6.2-3 This problem is pretty trivial, but the concept of the impedance of a shorted transmission line is important and used often. The impedance transformation formula is

\[ Z(l) = Z_0 \frac{Z_L \cos \beta l + jZ_0 \sin \beta l}{Z_0 \cos \beta l + jZ_L \sin \beta l} \]

If we set the load \((l = 0)\) impedance, \(Z_L\), to zero

\[ Z(l) = Z_0 \frac{jZ_0 \sin \beta l}{Z_0 \cos \beta l} = jZ_0 \tan \beta l \]

Then evaluate the impedance at \(l = L/2\) to get the equation (6-25) result.

\[ Z\left(\frac{L}{2}\right) = jZ_0 \tan \left(\beta \frac{L}{2}\right) \]
The easiest way to do this problem is with the array factor. We label the driven element, element zero, and the parasitic element, element one.

\[ AF(\theta) = I_0 e^{j\beta r_0 \hat{r}} + I_1 e^{j\beta r_1 \hat{r}} \]

Put the driven element at the origin and the parasitic element at \( z = d \). The unit vector has the usual expression.

\[
\begin{align*}
\mathbf{r}_0' &= 0 \\
\mathbf{r}_1' &= d\mathbf{\hat{z}} \\
\hat{r} &= \sin \theta \cos \phi \mathbf{\hat{x}} + \sin \theta \sin \phi \mathbf{\hat{y}} + \cos \theta \mathbf{\hat{z}}
\end{align*}
\]

The magnitude of the array factor toward and away from the parasitic element is

\[
\begin{align*}
|AF(0)| &= |I_0 + I_1 e^{j\beta d}| \\
|AF(\pi)| &= |I_0 + I_1 e^{-j\beta d}|
\end{align*}
\]

Plugging in the numbers we find

\[
\begin{align*}
I_{\text{driver}} &= 1 \angle 164^\circ & |AF(0)| &= 0.649 \\
I_{\text{parasitic}} &= 0.5 \angle 238^\circ & |AF(\pi)| &= 1.500 \\
\beta d &= \frac{2\pi}{\lambda} & 0.2\lambda &= 0.4\pi
\end{align*}
\]

Since the magnitude of the array factor is larger in the direction away from the parasitic element, the parasitic element is acting as a reflector. This can also be seen from a phasor diagram. These diagrams can vary depending on which phasor is added first, and what location is chosen for the origin. (I chose the location of the driver.)
6.4-2 Since there is no dissipation in the transmission line, and the load and transmission line are matched, the transmission line does not play a role in the solution. The efficiency is given by

\begin{equation}
    e = \frac{P_A}{P_A + P_t} = \frac{1}{2} |I|^2 R_0 + \frac{1}{2} |I|^2 R_t = -\frac{R_0}{R_0 + R_t}
\end{equation}

Simplifying, we have

\begin{equation}
    e = \frac{1}{1 + R_t / R_0}
\end{equation}

\begin{equation}
    R_t / R_0 = 1, 0.5, 0.1 \quad \Rightarrow \quad e = 0.50, 0.67, 0.91
\end{equation}