1. Write a numerical expression for $i(t)$, $t \geq 0$. 

Solin: Any $i(t)$ or $v(t)$ will be described by the general form of solution whenever we solve a switching problem:

$$i(t) = i(t^{-}) + [i(t^{-}) - i(t^{-})] \left[ 1 - e^{-t/RC} \right]$$

We only need to find $i(t^{-})$, $i(t^{+})$, and $RC$.

$i(t^{-})$: To find $i(t^{-})$, we need the initial conditions for the circuit. Since there is a capacitor, we need to find $v_c(t^{-})$. (If there were an inductor, we would need to find $i(t^{-})$.

Since the voltage on the capacitor cannot change instantly, $v_c(t^{-}) = v_c(t^{+})$.

To find $v_c(t^{+})$, we treat $C$ as an open circuit, (because the switch has been closed for a long time, allowing $C$ to charge to final value where current no longer flows).

Circuit model: $t = 0^{-}$ (switch closed)

Since no current flows in the 100Ω top right, $v_c(t^{-})$ also appears across the 200Ω R. (No current in 100Ω ⇒ no V drop.)
la) (cont) We have \( v_c(t=0^-) = \frac{3A \cdot 100V}{2} = 200V \)

\[ v_c(t=0^+) = v_c(t=0^-) \quad (v_c \text{ cannot change instantly}) \]

For \( t=0^+ \) we use a \( v \)-source to represent \( C \), and we find \( i(t=0^+) \):

\[ t=0^+ \quad \text{(switch open)} \]

We can find \( i(t) \) by any method we prefer. My choice is to take a Thevenin equivalent of the left side of the circuit, (thru the \( 200R \)).

\[ V_{Th} = \text{open circuit } v = 3A \cdot (100V + 100V) = 200V = 3A \cdot 100 \Omega = 300V \]

\[ R_{Th} = \text{resistance seen when looking into terminals } a-b \]

with independent \( 3A \) source set to zero. (This is the same as connecting a \( v \)-source at \( a-b \), measuring current into \( a \) and taking \( R_{Th} = v/I_a \).)

This simpler approach works whenever there are no dependent sources.

\[ u_{a-b} = R_{Th} \cdot i_{Th} = 100 \Omega \cdot (100V + 100V) = 200V \]

Our circuit is:

\[ t=0^+ \]

We see that \( v = 250V \), (the two \( 100R \)’s divide the \( 100V \) drop into two equal \( v \)-drops of \( 50V \) each).
1.a) (cont) Finally, we have \( i(t=0^+) = \frac{250V}{200\Omega} = 1.25A \uparrow \)

Note: I used a Thevenin equivalent that had \( i(t) \) transformed inside it, but I used it to find the output voltage \( V_o \) of the Thevenin equivalent. Then I found \( i(t=0^+) \) from the voltage. What I cannot do is try to find \( i(t) \) somewhere inside the Thevenin equivalent.

Note: We could use a mesh current, or node-\( V_o \) or superposition method to find \( i(t) \). User's choice. Superposition might be a good choice for finding \( i(t=0^+) \).

\( i(t=\infty) \): The capacitor will be charged and acting like an open circuit. Again, there will be no current thru the 100\( \Omega \) top right, and \( V(t=\infty) \) will appear across the 200\( \Omega \) R.

\[ \begin{align*}
&\text{100}\Omega \quad \text{3A} \quad \text{200}\Omega \quad i(t=\infty) \\
&\text{t}\to\infty \quad (\text{switch open})
\end{align*} \]

We have a current divider. \( i(t=\infty) = 3A \cdot \frac{100\Omega}{100\Omega+200\Omega} \)

\( i(t=\infty) = 1.5A \)

RC: Use the Thevenin equivalent found earlier for \( t=0^+ \) so we have simple RC circuit. Then \( R=R_0+100\Omega=200\Omega \).

\[ RC = 200\Omega \cdot 1\mu F = 200\mu \text{S} \]

\[ i(t=0^+) = 1.25A + (1.5-1.25A) \left(1-e^{-t/200\mu s} \right) = 1.5A - 0.25A \cdot e^{-t/200\mu s} \]

1.b) Calculate energy stored in \( C \) at \( t=0^+ \)

(Found in part 1.a)

\[ \text{Energy} = \frac{1}{2} C V^2(t=0^+) = \frac{1}{2} \cdot 1\mu F \cdot (200V)^2 = 0.02 \text{ J or 20 mJ} \]
1. Calculate energy stored in C as \( t \to \infty \).

Solve: \( w = \frac{1}{2} CV^2 = \frac{1}{2} \mu \text{F} \cdot (300 \text{V})^2 = 0.045 \text{J or 45 mJ} \).

Note: In part (a), we found \( V_C(t=\infty) = 300 \text{V} \).

2. Write an expression for \( V_1(t) \).

Solve: For \( t=0^- \), the \( L \) looks like a short circuit or wire.

Also, \( R_1 \) shorted by switch. \( V \)-divider gives \( V_1 \).

\[ V_1 = \frac{V_s}{R_1 + R_3} \cdot R_1 \quad t=0^- \]

With switch open, we can find \( i(t) \) through inductor.

We combine \( R' \).

\[ V_2 = \frac{V_s}{R_1 + R_2 + R_3} \cdot L \cdot i(t) \]

At \( t=\infty \), we again have \( L \) acting like a wire.

\[ i(t) = \frac{V_s}{R_1 + R_2 + R_3} \]

Now use general formula to write \( i(t) \).

\[ i(t) = i(t=0^+) + \left[ i(t=\infty) - i(t=0^+) \right] \left[ 1 - e^{-\frac{t}{L/R}} \right] \]

\[ i(t) = \frac{V_s}{R_1 + R_2 + R_3} \left[ 1 - e^{-\frac{t}{L/R}} \right] \]

\[ V_1(t) = i(t) \cdot R_1 \] since \( i(t) \) also flows through \( R_1 \).

\[ V_1(t) = V_s \cdot \left[ \frac{R_1}{R_1 + R_2 + R_3} + \left( \frac{R_1}{R_1 + R_2 + R_3} \right) - \frac{R_1}{R_1 + R_2 + R_3} \left[ 1 - e^{-\frac{t}{L/R}} \right] \right] \]
2.b) Make one consistency check (other than units).

**sol'n:** Check: If $V_g = 0$, I always discharged so $V(t = 0^+) = 0$.

Our answer is multiplied by $V_g$ so we get $V(t = 0^+) = 0$ if $V_g = 0$. √

or Check: If $R_1 = 0$, then $V_i = 0$ for short at $0^+$.

Our answer gives $V_i(t = 0^+) = V_g \left\{ \frac{0}{R_1} + \left[ \frac{0 - 0}{\left( R_3 + R_1 \right) R_3} \right] \right\} = 0$. √

**Note:** We have to be sure we don't get $0/0$ (which we have verified above, assuming $R_0, R_3 \neq 0$).

or Check: If $R_3 = 0$, then open switch doesn't add any $R_3$ and $I$ continues to act like wire, and $V_i = V_g R_1$.

Our answer gives $V_i(t = 0^+) = V_g \left\{ \frac{R_1}{R_3 + R_1} + \left[ \frac{R_1 - R_1}{(R_3 + R_1) R_3} \right] \right\}$

$= V_g \cdot \frac{R_1}{R_1 + R_3}$. √

or Check: If $R_0 = 0$, then we just have wire for $I$.

Then $V_i = V_g \cdot \frac{R_1}{R_3 + R_1}$ for $t = 0^+$.

Our answer gives $V_i(t = 0^+) = V_g \left\{ \frac{R_1}{R_3 + R_1} + \left[ \frac{R_1 - R_1}{(R_3 + R_1) R_3} \right] \right\} = \frac{-V_g R_1}{R_3 + R_1}$. √

Now, $e\to e = 0$.

$: V_i(t = 0^+) = V_g \left\{ \frac{R_1}{R_3 + R_1} + \frac{R_1 - R_1}{(R_3 + R_1) R_3} \right\} = \frac{-V_g R_1}{R_3 + R_1}$. √
a) Calculate the value of \( R_L \) that would absorb maximum power.

**Solution:** Transform into Thévenin equivalent of circuit left of \( R_L \).

Use superposition theorem and find \( V_{TH} \):

\[
V_{TH} = V_1 + V_2 \quad \text{In both cases, we have no current}
\]

\[
\text{through the top-right 100 \Omega}. \quad \text{So no V-drop across it.}
\]

\[
v_1 = \frac{V_0}{R_{TH}} \quad v_2 = 2A \times \frac{100 \Omega}{100 \Omega}
\]

\[
v_1 = 100V \times \frac{100 \Omega}{100 \Omega} = 50V
\]

\[
v_2 = 2A \times \frac{50 \Omega}{100 \Omega} = 100V
\]

\[
V_{TH} = V_1 + V_2 = 150V
\]

(Independent)

For \( R_{TH} \), we turn sources to zero and see what resistance we have looking in from \( a-b \) terminals:

\[
R_{TH} = 100 \Omega + 100 \Omega / 100 \Omega = 150 \Omega
\]

Max. power refer when \( R_L = R_{TH} = 150 \Omega \).
3.6) Calculate the value of max power, $P_L$, chosen for $max.p$

**Sol'n:** Our circuit with $R_L = R_{th}$ is:

![Circuit Diagram]

\[ \text{Power: } P = 150 \times 75 \times \left( \frac{150}{250} \right)^2 = 27.5 \text{ W} \]

4. Use superposition. Find expression for $v_2$ not using i.e.

**Sol'n:** Turn one independent source on at a time, with other independent sources = 0. Sum currents and $v$'s from each of the models to get total currents and voltages for circuit.

**Note:** Current source set to 0 is open circuit. Voltage $n-n$ is short circuit, (i.e., wire).

Circuit 1: $i_1 \neq 0, \ v_2 = 0$. Add a "1" subscript to $i_1$ and $v_2$.

**Use Node V for $v_{31}$:**

\[ i_2 = \frac{V_{21}}{R_1}, \quad i_{11} = \frac{V_{21}}{R_2}, \quad i_{31} = \frac{V_{31} - \beta V_{31}}{R_2} \]

\[ i_1 = v_2 \left( 1 + \frac{1}{R_1} - \frac{1}{R_2} \right) \text{ or } v_2 = i_1 \left( \frac{1}{R_1} + \frac{1}{R_2} - \frac{\beta}{R_1 R_2} \right) \]
4. (cont) Circuit 2: $V_s \neq 0, I_m = 0$. Add a "2" subscript to its $V$'s.

Use sum $V$'s around loop: $0$ (Mesh I)

$V_{22} = V_{32} = 0$

$V_2 = V_{30} \left( \frac{R_2 + 1 - \beta}{R_1} \right)$

Now $V_2 = V_{30} - \frac{V_{30} R_2 + \beta V_{32}}{R_1} + V_{32} = 0$

$V_3 = \frac{V_{30} \left( R_2 + 1 - \frac{\beta}{R_1} \right)}{R_1}$

$\beta = 0$, $V_3 = 0 \Rightarrow V_3 = i_g \left( R_{22} R_2 \right)$

Consistency Checks:

1. $\beta = 0$, $V_3 = 0 \Rightarrow V_3 = i_g \left( R_{22} R_2 \right)$

2. $R_1 = \infty \Rightarrow V_3 = V_2 + i_g R_2$

3. $I_s = 0$, $\beta = 0 \Rightarrow V_3 = V_2 \frac{R_1}{R_1 + R_2}$

4. $R_1 = 1 \Omega, R_2 = 2 \Omega, V_3 = 3V$, $i_s = 4A, V_{30} = 4V, \beta = 0.8$

$V_{30} = 4V, V_{32} = -(V_3 + \beta i_3) = 4 - (3 + 4 \times 0.8) = 4A \Rightarrow i_{R_2} = -4A$

$V_{30} = 4V, V_{32} = -(V_3 + \beta i_3) = 4 - (3 + 4 \times 0.8) = 4A \Rightarrow i_{R_2} = -4A$

$V_{30} = (4 + 4 \times 0.8) \times 0.8 = 8A, i_3 = 8A$