Ex: Give numerical answers to each of the following questions:

a) Rationalize $\frac{1-j\sqrt{3}}{1+j\sqrt{3}}$. Express your answer in rectangular form.

b) Find the polar form of $[(j+1)(-1-j)]^*$. (Note: the asterisk means "conjugate").

c) Find the following phasor: $P[-\sin(100t + 45^\circ)]$.

d) Find the magnitude of $\frac{(30-j40)}{(7-j24)e^{j30^\circ}}$.

e) Find the imaginary part of $\frac{6-j}{3j}$.

Sol'n: a) To rationalize, we multiply top and bottom by the conjugate of the denominator:

$$\frac{1-j\sqrt{3}}{1+j\sqrt{3}} = \frac{1-j\sqrt{3}}{1+j\sqrt{3}} \cdot \frac{1-j\sqrt{3}}{1-j\sqrt{3}} = \frac{1-3-j2\sqrt{3}}{1^2 + \sqrt{3}^2} = \frac{-2-j2\sqrt{3}}{4}$$

or

$$\frac{1-j\sqrt{3}}{1+j\sqrt{3}} = \frac{-1}{2} - \frac{j\sqrt{3}}{2}$$

b) The conjugate is obtained by changing $j$ to $-j$ throughout the expression:

$$[(j+1)(-1-j)]^* = (-j+1)(-1+j) = (1-j)(1+j)$$

Converting the rectangular forms to polar forms before multiplying yields an answer in polar form:

$$(1-j)(1+j) = \sqrt{2} \angle -45^\circ \cdot \sqrt{2} \angle 135^\circ = 2 \angle 90^\circ$$

c) The phasor of $\sin(\omega t)$ is $-j$:

$$P[-\sin(100t + 45^\circ)] = -(-j) \cdot 1 \angle 45^\circ = j \cdot 1 \angle 45^\circ$$

or

$$P[-\sin(100t + 45^\circ)] = 1 \angle 90^\circ \cdot 1 \angle 45^\circ = 1 \angle 45^\circ$$
d) The magnitude of a product (or quotient) is the product (or quotient) of the magnitudes:

\[
\left| \frac{(30 - j40)}{(7 - j24)e^{j30^\circ}} \right| = \left| 30 - j40 \right| \left| 7 - j24 \right| e^{j30^\circ} = \frac{\sqrt{30^2 + 40^2}}{\sqrt{7^2 + 24^2}} = \frac{\sqrt{900 + 1600}}{\sqrt{49 + 576}} = \frac{\sqrt{2500}}{\sqrt{625}} = 2
\]

or

\[
\left| \frac{(30 - j40)}{(7 - j24)e^{j30^\circ}} \right| = \frac{50}{25} = 2
\]

e) Note that the imaginary part is a real number:

\[
\text{Im}\left[ \frac{6 - j}{3j} \right] = \text{Im}\left[ \frac{-j(6 - j)}{3} \right] = \text{Im}\left[ \frac{-1 - j6}{3} \right] = \text{Im}\left[ \frac{-1}{3} - j2 \right] = -2
\]