Ex:

\[ \begin{align*}
R_1 \downarrow i_1 & \quad R_2 \downarrow i_a \\
\downarrow i_a & \quad R_3 \\
\end{align*} \]

Derive an expression for \( i_1 \). The expression must not contain more than the circuit parameters \( v_a, i_a, R_1, R_2, \) and \( R_3 \).

Sol’N: Using Kirchhoff’s and Ohm’s laws, we write equations for the circuit. The first step is to label the current and voltage for each resistor in accordance with the passive sign convention.

The labeling of voltages and currents may be done in either of two directions, so long as the arrow of the current measurement points toward the minus sign of the voltage measurement. Placing + signs at the left or on top usually works well, however.

\[ \begin{align*}
R_1 \downarrow i_1 v_1 & \quad + \quad R_2 \downarrow i_2 v_2 \\
\downarrow i_a & \quad R_3 \\
\end{align*} \]

By Ohm's law, resistor voltages may be written in terms of resistor currents:
\[ v_1 = i_1 R_1 \]
\[ v_2 = i_2 R_2 \]
\[ v_3 = i_3 R_3 \]

If we use \( i \cdot R \) in place of \( v \), we obtain formulas in terms of only currents. This reduces the final number of equations we will generate.

Starting with \( v \)-loops, the only \( v \)-loop we use is the outer loop, as all other loops would require us to define a voltage for current source \( i_a \).

For the outer loop, traveling clockwise from the lower left yields the following:
\[ i_3 R_3 + i_1 R_1 - v_a - i_2 R_2 = 0 \text{ V} \]  
(1)

Now we consider currents flowing out of nodes, and we consider currents for components in series.

\( R_1 \), \( v_a \), and \( R_2 \) are in series and must carry the same physical current. Given that the direction of current measurement for \( i_2 \) is opposite to the direction of current measurement for \( i_1 \), we have the following equation:
\[ i_2 = -i_1 \]  
(2)

Summing currents out of the node on the left yields the following equation:
\[ -i_1 + i_a + i_3 = 0 \text{ A} \]  
(3)

Summing currents out of the node on the right yields the same equation multiplied by \(-1\), (after we substitute \(-i_1 \) for \( i_2 \)). Thus, we omit the equation for the right node.

From equation (3), we have an expression for \( i_3 \) in terms of \( i_1 \) and \( i_a \):
\[ i_3 = i_1 - i_a \]

Using this result and equation (2) in equation (1) allows us to write an equation involving \( i_1 \) as the only unknown:
\[ (i_1 - i_a) R_3 + i_1 R_1 - v_a + i_1 R_2 = 0 \text{ V} \]
or

\[ i_1 (R_1 + R_2 + R_3) = v_a + i_a R_3 \]

or

\[ i_1 = \frac{v_a + i_a R_3}{R_1 + R_2 + R_3} \]