Ex:

a) Use the node-voltage method to calculate \( v_1 \) and \( i_2 \).

b) Calculate the power in the 1.5 k\( \Omega \) resistor.

**Solution:**

a) We first assign a reference node. The node on the left is convenient since it is connected to the - of the 1.8V supply.

We assign node voltages \( v_2 \) and \( v_3 \) on the right side of the circuit, (since \( v_1 \) is already used).
We check for dependent sources and super nodes, but neither is present in this circuit.

Now we write current summation eqns for each node.

\[ v_2 \text{ node: } -4 \, mA + \frac{v_2 - 1.8V + v_2 - v_3}{300 \Omega} + \frac{v_2 - v_3}{250 \Omega} = 0A \]

\[ v_3 \text{ node: } \frac{v_3 - v_2}{250 \Omega} + \frac{v_3 - 0V + v_3 - 0V}{1.5 \, k\Omega} + \frac{v_3 - 0V}{7.5 \, k\Omega} = 0A \]

We solve these eqns for \( v_2 \) and \( v_3 \).

Grouping terms multiplying \( v_2 \) and \( v_3 \) and putting constant terms on the right side of the eqn keeps things organized.

\[ v_2 \left( \frac{1}{300 \Omega} + \frac{1}{250 \Omega} \right) + v_3 \left( \frac{-1}{250 \Omega} \right) = \frac{4 mA + 1.8V}{300 \Omega} \]

\[ v_2 \left( \frac{-1}{250 \Omega} \right) + v_3 \left( \frac{1}{250 \Omega} + \frac{1}{1.5 \, k\Omega} + \frac{1}{7.5 \, k\Omega} \right) = 0A \]

Multiplying both sides by 7.5k\( \Omega \) clears the denominators.

\[ v_2 \left( 25 + 30 \right) + v_3 \left( -30 \right) = 30V + 1.8V \left( 25 \right) \]

\[ v_2 \left( -30 \right) + v_3 \left( 30 + 5 + 1 \right) = 0V \]

The 2nd eqn is easier to solve. (We may solve for either \( v_2 \) or \( v_3 \).)
\[ v_2 = v_3 \cdot \frac{36}{30} = v_3 \cdot \frac{6}{5} \]

Substituting into the 1st eqn, we have

\[ v_3 \cdot \frac{6}{5} \cdot (55) + v_3 (-30) = 30V + 45V = 75V \]

or \[ v_3 \cdot (66 - 30) = 75V \]

or \[ v_3 \cdot 36 = 75V \]

or \[ v_3 = \frac{75}{36} \cdot V = \frac{25}{12} \cdot V \]

Using or earlier eqn for \( v_2 \), we have

\[ v_2 = v_3 \cdot \frac{6}{5} = \frac{25}{12} \cdot \frac{6}{5} = \frac{5}{2} \cdot V \]

Before going further we perform a consistency check on the currents to verify that they sum to zero at each node:

Check: \(-4mA + \frac{7}{3} mA + \frac{5}{3} mA = 0 \ mA \checkmark \)

\(-\frac{5}{3} mA + \frac{25}{18} mA + \frac{25}{90} mA = 0 \ mA \checkmark \)
For \( v_1 \) we have

\[ V_1 = v_2 - v_3 = \frac{5}{2} - \frac{2.5}{12} \ V \]

or

\[ V_1 = \frac{30 - 2.5 \ V}{12} = \frac{5}{12} \ V \]

For \( i_2 \) we have

\[ i_2 = \frac{v_3}{7.5 \Omega} = \frac{2.5/12}{7.5} \ mA = \frac{5}{18} \ mA \]

(Note: \( v_1 \) and \( i_2 \) were actually found earlier in the consistency check.)

(b) The power in the 300 \( \Omega \) resistor is

\[
p = v \cdot i = (v_2 - 1.8v) \cdot (v_2 - 1.8v) \cdot \frac{300 \Omega}{300} = \frac{(2.5v - 1.8v)^2}{300} = \frac{(0.7)^2}{300} W
\]

\[ p \cong 1.63 \ mW \]

The power in the 1.5 k\( \Omega \) resistor is

\[
p = v \cdot i = \frac{(v_2 - v_1)^2}{1.5 \ k\Omega} = \frac{(5 - 5 \frac{5}{12})^2}{1.5 \ k\Omega} = \frac{(25 \frac{25}{12}^2}{1.5 \ k\Omega} = 2.89 \ mW
\]