Ex:

Use the mesh-current method to find $i_1$, $i_2$, and $i_3$. 

**SOL’N:** The mesh-current method is based on the idea of calculating voltage loops in terms of mesh currents in inner loops that add up to give the total current through components that are shared by loops. The diagram below shows the polarities of voltage drops for mesh currents. These voltage drops follow the passive sign convention for their respective mesh current.

The current through the $2\Omega$ resistor (measured with an arrow pointing down) is $i_1 - i_2$. The current through the $3\Omega$ resistor (measured with an arrow pointing down) is $i_2 - i_3$.

On the right side, we have a current source on the outside edge of the loop. It follows that mesh current, $i_3$, is determined by the current of the source:

$$i_3 = 5 \text{ A}$$

For the other loops, we use voltage loops. The voltage loop for mesh current $i_1$ is as follows:

$$12 \text{ V} - i_1 \cdot 2 \Omega + i_2 \cdot 2 \Omega = 0 \text{ V}$$  (1)
The voltage loop for mesh current $i_2$ is as follows:

$$-i_2 \cdot 2 \, \Omega + i_1 \cdot 2 \, \Omega - i_2 \cdot 3 \, \Omega + i_3 \cdot 3 \, \Omega - 15 \, V = 0 \text{V} \quad (2)$$

We substitute $i_3 = 5 \, A$ and solve the simultaneous equations by first combining terms multiplying $i$'s and moving constant terms to the right side.

$$-i_1 \cdot 2 \, \Omega + i_2 \cdot 2 \, \Omega = -12 \, V \quad (1')$$

$$i_1 \cdot 2 \, \Omega - i_2 \cdot (2 \, \Omega + 3 \, \Omega) + 5 \, A \cdot 3 \, \Omega = 15 \, \text{V}$$

or

$$i_1 \cdot 2 \, \Omega - i_2 \cdot (2 \, \Omega + 3 \, \Omega) = 15 \, \text{V} - 5 \, A \cdot 3 \, \Omega = 0 \, \text{V} \quad (2')$$

Summing the two equations eliminates the $i_1$ term.

$$i_2 \cdot 2 \, \Omega - i_2 \cdot (2 \, \Omega + 3 \, \Omega) = -12 \, \text{V} + 0 \, \text{V} = -12 \, \text{V} \quad (3)$$

or

$$-i_2 \cdot 3 \, \Omega = -12 \, \text{V}$$

or

$$i_2 = 4 \, \text{A}$$

Using equation 1' and the value of $i_2$, we find the value of $i_1$:

$$-i_1 \cdot 2 \, \Omega + i_2 \cdot 2 \, \Omega = -12 \, \text{V}$$

or

$$-i_1 \cdot 2 \, \Omega = -12 \, \text{V} - i_2 \cdot 2 \, \Omega = -12 \, \text{V} - 4 \, \text{A} \cdot 2 \, \Omega = -20 \, \text{V}$$

or

$$i_1 = \frac{20 \, \text{V}}{2 \, \Omega} = 10 \, \text{A}$$