Ex:

Make a consistency check on the following equations for the above circuit by setting resistors and sources to values for which the values of $v_1$, $v_2$, and $v_3$ are obvious. State the values of resistors, sources, and node voltages for your consistency check, and show that the circuit equations are satisfied for these values. (In other words, plug the values into the equations and show that the left side and the right side of each equation are equal.)

$$v_s = v_1 - v_2$$

$$-i_s + \frac{v_1}{R_3} + \frac{v_2 - v_3}{R_2} + \frac{v_2 - v_3}{R_5} + \frac{v_2}{R_4} = 0 \text{ A}$$

$$i_s + \frac{v_3 - v_2}{R_2} + \frac{v_3 - v_2}{R_5} - \alpha \frac{v_2 - v_3}{R_5} = 0 \text{ A}$$

Sol’N: Many checks are possible. One example is given here.

Suppose $i_s = 0 \text{ A}$, $\alpha = 0$, $v_s = 7 \text{ V}$, $R_1 = 1 \Omega$, $R_2 = 2 \Omega$, $R_3 = 3 \Omega$, $R_4 = 4 \Omega$, and $R_5 = 5 \Omega$. The resulting circuit is shown below. We see that turning off the two current sources causes $R_1$, $R_2$, and $R_5$ to dangle at the end of wires. No current flows in these resistors, as there is no complete circuit. By Ohm's law, it follows that there is no voltage drop across these wires, implying that $v_3 = v_2$. We also have $v_1 = v_2 + 7 \text{ V}$ owing to the $v_s = 7 \text{ V}$ source.
We are left with the problem of finding $v_2$. For that, we observe that we have a voltage divider consisting of $v_s$, $R_3$, and $R_4$. Being careful to use the correct sign for voltage $v_2$ we have the following result:

\[ v_2 = -v_s \frac{R_4}{R_3 + R_4} = -7 \text{V} \frac{4 \Omega}{3 \Omega + 4 \Omega} = -4 \text{V} \]

Using earlier equations, we have the following node voltages:

\[ v_1 = 3 \text{V}, \ v_2 = -4 \text{V}, \text{ and } v_3 = -4 \text{V} \]

Now we plug in numerical values for all the terms in the node-voltage equations and check that the two sides are equal:

\[ v_s = v_1 - v_2 \quad \text{or} \quad 7 \text{V} = 3 \text{V} - (-4 \text{V}) \quad \sqrt{\text{checks out}} \]

\[ -i_s + \frac{v_1}{R_3} + \frac{v_2 - v_3}{R_2} + \frac{v_2 - v_3}{R_5} + \frac{v_2}{R_4} = 0 \text{A} \quad \text{or} \quad -0\text{A} + \frac{3\text{V}}{3\Omega} + \frac{-4\text{V} - (-4\text{V})}{2\Omega} + \frac{-4\text{V} - (-4\text{V})}{5\Omega} + \frac{-4\text{V}}{4\Omega} = 0 \text{A} \quad \sqrt{\text{checks out}} \]

\[ i_s + \frac{v_3 - v_2}{R_2} + \frac{v_3 - v_2}{R_5} - \alpha \frac{v_2 - v_3}{R_5} = 0 \text{A} \quad \text{or} \quad 0\text{A} + \frac{-4\text{V} - (-4\text{V})}{2\Omega} + \frac{-4\text{V} - (-4\text{V})}{5\Omega} - 0\frac{-4\text{V} - (-4\text{V})}{5\Omega} = 0 \text{A} \quad \sqrt{\text{checks out}} \]