**Ex:** Find the voltage, \( v_C \), on the capacitor in the circuit below as a function of time if \( v_C(t = 0^+) = 6 \) V.

\[
\begin{align*}
C &= 0.5 \, \mu\text{F} \\
R &= 100 \, \text{k}\Omega
\end{align*}
\]

**SOL’N:** We observe that voltage \( v_C \) appears across both the \( R \) and \( C \), as shown below.

\[
\begin{align*}
C &= 0.5 \, \mu\text{F} \\
R &= 100 \, \text{k}\Omega
\end{align*}
\]

Using the defining equation of a capacitor and Ohm's law, we have the following results:

\[
i_C = C \frac{dv_C}{dt}
\]

and

\[
i_R = \frac{v_C}{R}
\]

Since the current must be the same everywhere in the loop, and since the currents are measured with opposite polarities, we have that \( i_C \) and \( i_R \) are equal but opposite.

\[
i_C = -i_R
\]

or

\[
C \frac{dv_C}{dt} = -\frac{v_C}{R}
\]

One way to solve this equation is to separate the variables:

\[
C \frac{1}{v_C} dv_C = -\frac{1}{R} dt
\]
Integrating both sides yields the following result:

\[ C \int_{v_C(t=0)}^{v_C(t)} \frac{1}{v_C} dv_C = -\frac{1}{R} \int_0^t dt \]

or

\[ C \ln v_C(t) \big|_{v_C(t=0)}^{v_C(t)} = -\frac{1}{R} t \]

or

\[ C \left[ \ln v_C(t) - \ln v_C(0) \right] = -\frac{1}{R} t \]

or

\[ \ln \frac{v_C(t)}{v_C(0)} = -\frac{t}{RC} \]

or

\[ v_C(t) = v_C(0) e^{-\frac{t}{RC}} \]

or

\[ v_C(t) = v_C(0) e^{\frac{t}{RC}} \]

Substituting values given in the problem, we have the following answer:

\[ v_C(t) = 6V \cdot e^{-\frac{t}{100k\Omega \cdot 0.5\mu F}} = 6V \cdot e^{-\frac{t}{50ms}} \]