In the above circuit, the switch moves from a to b at t = 0.

1. Find values of R and C such that the roots of the characteristic equation of the circuit for $t > 0$ are $s_1 = -800 + j600$ and $s_2 = -800 - j600$ r/s.

2. Given that no energy is stored in the inductor at time $t = 0$, find a numerical expression for $v(t)$ for $t > 0$.

**Soln:**

1) After $t = 0$ we have parallel RLC circuit.

$$
\begin{align*}
\frac{s^2}{RC} + \frac{s}{LC} + 1 &= 0, \\
\frac{1}{2RC} \ln \left( \frac{s + \frac{1}{RC}}{\frac{1}{2RC} - s} \right) &= 0, \\
\frac{1}{2RC} &= 300 \text{ r/s}, \\
\frac{1}{2RC} &= \frac{1}{2 \cdot 2\cdot 800 \cdot 10 \mu F}, \\
R &= \frac{1}{2RC}, \\
C &= \frac{1}{L^2} = \frac{1}{10 \mu F^2} = 10 \mu F.
\end{align*}
$$

R = 1/2 RC

\[ R = \frac{1}{2 \cdot 800 \cdot 10 \mu F} \]

\[ R = \frac{1}{16 \cdot 1 \mu F^2 / s} \]

\[ R = 62.5 \Omega \]

\[ C = 10 \mu F \]

\[ R = \frac{1}{2RC} \]

2) Open wire (no current in L, discharges thru

Therein equiv of 10 mA 1/10mA. 1kΩ 2X

\[ \text{At } t = 0^+ : \quad v_0(t) = v_c(0^+) = v_c(0^-) = 10V \]

\[ \frac{dv(t)}{dt} \bigg|_{t=0^+} = \frac{i_c(0^+)}{C} + i_L(0^+), \quad i_c(0^+) + i_L(0^+) = 0A \]

\[ i_L(0^+) = i_L(0^-) = 0A \]

\[ i_R(0^+) = \frac{v_C(0^+)}{R} = \frac{10V}{R} \]

\[ i_L(0^+) = \frac{-160mA}{16 \mu F} \]

\[ i_L(0^+) = -160mA, \quad 16 \text{ kV/s} \]

\[ \therefore i_C(0^+) = -160mA \quad \frac{dv(t)}{dt} \bigg|_{t=0^+} = -160mA \]
sol(n: 2) cont. \[ v(t) = A_1 e^{-\frac{t}{600}} \cos \omega_d t + A_2 e^{-\frac{t}{600}} \sin \omega_d t, \quad t > 0 \]

\[ \frac{dv(t)}{dt} \bigg|_{t=0^+} = A_2 \omega_d \quad \Rightarrow \quad A_1 = -1 \text{ kV/s} \]

\[ \omega_d = 600 \text{ rad/s} \quad \eta = 800 \]

\[ v(0^+) = A_1 - 10 \text{ V} \]

\[ A_1 = 10 \text{ V} \]

\[ A_2 = 600 \cdot 800 - 800 \cdot 10 \text{ V} = -1 \text{ kV/s} \]

\[ A_2 = \frac{-16k + 8k}{0.6k} = \frac{-8k}{0.6k} = -\frac{80}{6} = -13\frac{1}{3} \text{ V} = -40 \frac{1}{3} \text{ V} \]

\[ v(t) = 10 e^{-\frac{t}{600}} \cos 600t - \frac{40}{3} e^{-\frac{t}{600}} \sin 600t \text{ V} \]
3.

\begin{center}
\includegraphics[width=0.5\textwidth]{circuit.png}
\end{center}

\[ T = \text{one period of } v_i(t) = 3\pi \mu s \]

a. Find values of \( C \) and \( L \) for the above filter circuit such that the transfer function equals one for the fundamental frequency and zero for the third harmonic of \( v_i(t) \), also shown above.

b. Find numerical values of coefficients \( a_0, a_1, a_2, b_1, \) and \( b_2 \) for the Fourier series for \( v_i(t) \):

\[ v_i(t) = a_0 + \sum_{k=1}^{\infty} a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \]

\[ \text{Solution: a) } \omega_1 = \frac{2\pi}{T} = \frac{2\pi}{3\pi\mu s} = \frac{2}{3} \text{ M/}\mu s \]

\[ \omega_2 = 3\omega_1 = 2 \text{ M/}\mu s \]

\[ \omega_k \text{ for transfer function when } \omega \text{ across rails } = 0 \text{ rad}, \]

i.e., when \( \frac{1}{sC} + sL_1 = 0 \), \( L_1 = 10 \mu H \), or \( \omega = \frac{1}{L_1 C} \)

and when \( \left( \frac{1}{sC} + sL_1 \right) sL_2 = \omega \), we get transfer func = 1

2nd condition: \( \left( \frac{1}{sC} + sL_1 \right) sL_2 = \frac{L_2 + s^2 L_1 L_2}{sC + s(L_1 + L_2)} \)

\[ u = \frac{L_1 L_2}{L_1 + L_2} \cdot \frac{s^2 + \frac{1}{L_1 C}}{s^2 + \frac{1}{(L_1 + L_2) C}} = \infty \]
\[ \text{Soln: 3a) cont. } \]

2nd condition gives \( \infty \) when denominator = 0:

\[ \frac{s^2 + \frac{1}{(L_1 + L_2)C}}{0} = 0 \]

or \( \omega_1^2 = \frac{1}{L_1(C + 1)} \)

Back to first condition: \( \omega_3^2 = \frac{1}{L_1 C}, \quad C = \frac{\omega_3^2}{L_1} \)

\[ C = \frac{\frac{1}{10 \mu (2 \, \text{M}^2)^2}}{40} = \frac{25 \mu F}{40 \cdot 25} = 25 \text{nF} \]

\[ C = 25 \text{nF} \]

2nd condition: \( L_1 + L_2 = \frac{1}{\omega_3^2 C} = \frac{1}{(\frac{2}{3})^2 \cdot 25 \text{nF}} \times \frac{\lambda \text{mH}}{100} \)

\[ \lambda = 90 \, \text{mH} \]

\[ L = L_2 = 80 \mu \text{H} \]

b) \( a_v = \text{ave value of } v_i(t) = 4 \text{V} \) by inspection, \( a_v = 4 \text{V} \)

\( v_i(t) \) has shift-flip symmetry \( \Rightarrow \text{even terms} = 0 \).

\( a_0 = 0 \quad b_0 = 0 \)

\[ a_1 = \frac{2}{T} \int_0^T v_i(t) \cos(\omega_0 t) \, dt = \frac{2}{T} \int_0^{T/2} \left( \frac{4}{T} + \frac{4 \pm T/2}{T} \right) \cos(\omega_0 t) \, dt \]

Subtract DC offset \( a_0 = 4 \text{V} \) from \( v_i(t) \), \[ \text{doesn't change } a_1 \]

\[ a_1 = \frac{4}{T} \int_0^{T/2} \cos(\omega_0 t) \, dt, \quad \omega_0 = \frac{2\pi}{T} \]

Using table of integrals:

\[ a_1 = \frac{4}{T^2} \left[ \frac{1}{\omega_0^2} \cos(\omega_0 t) + \frac{t}{\omega_0} \sin(\omega_0 t) \right]_0^{T/2} \]

Note \( \omega_0 \cdot \frac{T}{2} = \frac{2\pi}{T} \cdot \frac{T}{2} = \pi \)

\[ \cos \pi = 1 \quad \cos \frac{\pi}{2} = 0 \]

\[ \sin \pi = 0 \quad \sin \frac{\pi}{2} = 1 \]

\[ a_1 = \frac{32}{T^2} \cdot \frac{\omega_0^2}{(2\pi)} \cdot (1 - (1)) = \frac{-32 \cdot \frac{2}{4\pi^2}}{\frac{16}{\pi^2}} = -\frac{16}{\pi^2} \]

\[ a_1 = -\frac{16}{\pi^2} \]
sol'n: 3.b) cont. \[ b_1 = \frac{4}{T} \int_0^{T/2} \frac{4}{T/2} \sin(w_0 t) \, dt \]

\[ b_1 = \frac{4^2}{T^2} \left[ \left. \frac{1}{w_0^2} \left( \frac{1}{2} \sin w_0 t - \frac{1}{w_0} \cos w_0 t \right) \right|_0^{T/2} \right] \]

\[ = \frac{32}{T^2 (2\pi)} \left[ -\frac{1}{2} (-1) \right] = \frac{32}{4\pi} = \frac{8}{\pi} \]

\[ b_1 = \frac{8}{\pi} \]

Note: only the \( \frac{1}{2} \) cos term is nonzero.