1. Consider a traditional BJT current mirror with a nominal current transfer ratio of unity. (e.g., $I_o/I_{ref} = 1$) $I_S = 10^{-15}$ A, $\beta = 100$, and $V_A = 100$ V. For $I_{ref} = 1$ mA, find $I_O$ when $V_o = 5$ V. Also, find the output resistance.

$$I_o = I_{ref} \left( \frac{m}{1 + \frac{m - 1}{\beta}} \right) \left( 1 + \frac{V_a - V_{BE}}{V_{A2}} \right)$$

$$I_o = 1 \text{ mA} \left( \frac{1}{1 + \frac{1}{100}} \right) \left( 1 + \frac{5 - 0.7}{100} \right) = 1.02 \text{ mA}$$

$$R_o = \frac{V_a}{I_o} = \frac{100 \text{ V}}{1.02 \text{ mA}} = 98 \text{ k}\Omega \approx 100 \text{ k}\Omega$$

2. Assuming the availability of BJTs with scale currents $I_S = 10^{-15}$ A, $\beta = 100$, and $V_A = 50$ V, design the current-source circuit of the figure below to provide an output current $I_o = 0.5$ mA at $V_o = 2$ V. Use a power supply $V_{CC} = 5$ V. Give the values of $I_{ref}$, $R$, and $V_{OMIN}$. Also, find the actual value of $I_O$ at $V_o = 5$ V.

$$I_o = \frac{I_{ref}}{1 + (2/\beta)} \left( 1 + \frac{V_o - V_{BE}}{V_A} \right)$$

$$0.5 \text{ mA} = \frac{I_{ref}}{1 + (2/100)} \left( 1 + \frac{2 - 0.7}{50} \right) \Rightarrow I_{ref} = 0.497 \text{ mA}$$

$$I_{ref} = \frac{V_{CC} - V_{BE}}{R} \Rightarrow R = \frac{V_{CC} - V_{BE}}{I_{ref}}$$

$$R = \frac{5 - 0.7}{0.497 \text{ mA}} = \frac{4.3}{0.497} = 8.65 \text{ k}\Omega$$

$$V_{OMin} = V_{CESAT} = 0.3 \text{ V}$$

For $V_o = 5$ V:

$$I_o = \frac{I_{ref}}{1 + (2/\beta)} \left( 1 + \frac{V_o - V_{BE}}{V_A} \right)$$

$$I_o = \frac{0.497}{1 + (2/100)} \left( 1 + \frac{5 - 0.7}{50} \right) = 0.53 \text{ mA}$$

3. Find the output resistance of each of the two current sources below. Let $R_1 = 942 \text{ k}\Omega$, $R_2 = 9.3 \text{ k}\Omega$, $R_3 = 11.5 \text{ k}\Omega$, $V_A = 100$ V and $\beta = 100$.

$$\delta_m = \frac{I_C}{V_T} = \frac{10 \mu A}{25 \text{ mV}} = 0.4 \text{ mA/V} \text{ and}$$

$$r_o = \frac{V_s}{I_C} = \frac{100 \text{ V}}{10 \mu A} = 10 \text{ M}\Omega, \quad r_m = \frac{\beta}{\delta_m} = 250 \text{ k}\Omega$$

For (a) $r_o = r_m = r_o = 10 \text{ M}\Omega$

For (b) $r_o \approx \frac{1 + \delta_m (R_E \parallel R_m)}{r_o}$

$$R_o \approx \left[ 1 + 0.4 \left( \frac{\text{mA}}{V} (11.5 \text{ k}\Omega \parallel 250 \text{ k}\Omega) \right) \right] 10 \text{ M}\Omega$$

$$r_o = 54 \text{ M}\Omega$$
4. The current-source circuit of the figure to the right utilizes a pair of matched pnp transistors having $I_S = 10^{-15}$ A, $\beta = 100$, and $V_T = 50$ V. It is required to design the circuit to provide an output current $I_o = 1$ mA at $V_o = 2$ V. What values of $I_{REF}$ and $R$ are needed? What is the maximum allowed value of $V_o$ while the current source continues to operate properly? What change occurs in corresponding $I_o$ to $V_o$ changing from the maximum positive value to -5 V?

\[
I_o = I_S \frac{V_{BE}}{V_T} \\
\left| I_m \right| = 10^{-15} e^{\frac{V_{BE}}{25m}} \\
\left| V_{BE} \right| = 25m \ln \left( \frac{1m}{10^{-15}} \right) = .69\text{ V} \\
\frac{I_o}{I_{REF}} = \frac{1}{1 + \left( \frac{2}{\beta} \right)} \left[ 1 + \frac{\left| V_o - V_{BE} \right|}{\left| V_T \right|} \right] \\
\left| I_m \right| = \frac{I_{REF}}{1 + \left( \frac{2}{\beta} \right)} \left[ 1 + \frac{2 - 0.69}{50} \right] \\
I_{REF} = \frac{1.02m}{1.0262} = 0.994\text{ mA} - I_{REF} \\
5 - 0.7 - I_{REF} R = 0 \\
R = \frac{4.3}{0.994\text{ m}} = 4.3\text{ k}\Omega \\
\text{max } V_o \text{ when } V_c = V_B = 5 - 0.7 = 4.3\text{ V} \\
I_o = \frac{0.994\text{ m}}{1 + \frac{2}{100}} \left[ 1 + \frac{5 - 0.69}{50} \right] = 1.09\text{ mA} \]
5. Find the voltages at all nodes and the currents through all branches in the circuit below. Assume \( |V_{BE}| = 0.7 \text{V} \) and \( \beta = \infty \).

\[
I_{C1} = I_{C2} = I_{R1} \\
V_{B1} = V_{CC} - V_{EB} = 10 - 0.7 = 9.3 \text{ V} \\
V_{B2} = V_{EE} + V_{BE} = -10 + 0.7 = -9.3 \text{ V} \\
I_{R1} = \frac{V_{B1} - V_{B2}}{R_1} = \frac{9.3 - (-9.3)}{20 \text{ k}\Omega} = 0.93 \text{ mA} \\
I_{C1} = I_{C2} = I_{C3} = I_{C4} = I_{C5} = I_{C6} = 0.93 \text{ mA} \\
V_{C3} = 0.93 \text{ mA} (5 \text{ k}\Omega) = 4.65 \text{ V} \\
V_{C5} = V_{BE5} = 0.7 \text{ V} \\
V_{C6} = 5 \text{ V} - 0.93 \text{ mA} (3.6 \text{ k}\Omega) = 1.65 \text{ V} \\
I_{C9} - I_{C8} - I_{C7} - I_{C2} = 0.93 \text{ mA} \\
I_{R4} = 2 (0.93 \text{ mA}) = 1.86 \text{ mA} \\
V_{C7} = V_{C8} = 0 - I_{R4} R_4 \\
= 0 - 1.86 \text{ mA} (2 \text{ k}\Omega) = -3.72 \text{ V} \\
I_{C11} = I_{C10} = I_{C9} = 0.93 \text{ mA} \\
V_{C9} = V_{C10} = V_{P10} = 5 - 0.7 = 4.3 \text{ V} \\
V_{C11} = 0.93 \text{ mA} (3 \text{ k}\Omega) = 2.79 \text{ V}
6. Using the ideas embodied in the figure below, design a multiple-mirror circuit using power supplies of ±5V to create source currents of 0.4mA, and 0.8mA (currents shown below as $I_{\text{REF}}$, $I_1$, and $I_3$) and sink currents of 1mA, and 2mA (currents shown below as $I_2$, and $I_4$). Assume that the BJTs have $|V_{BE}| = 0.7$ and large $\beta$. 

\[ R = \frac{4.3 - (4.3)}{0.2m} = 43K \Omega \]

Approximate power =

\[ +5(0.2m + 0.4m + 0.8m) + 5(0.2m + 1m + 2m) \]

\[ P \approx 23mW \]
7. Find the output resistance of the double-cascode current mirror below by drawing the hybrid pi model and finding the equivalent resistance at the drain of Q3. (Hint: Use a test source and find the Thevenin equivalent resistance).

\[
\text{Node at } V_{at 1}: \quad -g_{m5}V_{gs5} + g_{m6}V_{gs6} + g_{m3}V_{gs3} + \frac{V_{gs5}}{R_{o5}} = 0 \\
V_{gs5} \left( g_{m5} + \frac{1}{R_{o5}} \right) = 0 \Rightarrow V_{gs5} = 0
\]

\[
\text{Node at } V_{at 2}: \quad -g_{m4}V_{gs4} - \frac{V_{gs5}}{R_{o5}} + g_{m4}V_{gs4} + \frac{V_{gs4}}{R_{o4}} = 0 \\
V_{gs4} \left( g_{m4} + \frac{1}{R_{o4}} \right) = 0 \Rightarrow V_{gs4} = 0
\]

Note that the current through \( R_{o6} \) is \( g_{m6}V_{gs6} \)

\[
+V_{gs6} + V_{gs6}g_{m6}(R_{o6}) = 0 \\
V_{gs6} \left( 1 + g_{m6}(R_{o6}) \right) = 0 \Rightarrow V_{gs6} = 0
\]

All the diode connected transistor become an equivalent ground.

\[
V_{gs1} = 0
\]
Redrawing knowing eq. R:

Node - V at 1:

\[ -I_{test} + g_{m3}V_{gs} + \left(1 + \frac{V_{gs}}{R_{03}}\right) = 0 \]

\[ \therefore I_{test} = V_{gs3}(g_{m3} + \frac{1}{R_{03}}) + \frac{1}{R_{03}} \]

Node - V at 2:

\[ -g_{m3}V_{gs3} + \left(\frac{V_{gs3} - 1}{R_{03}}\right) + \frac{V_{gs}}{R_{eq}} = 0 \]

\[ V_{gs3}\left(g_{m3} + \frac{1}{R_{03}} + \frac{1}{R_{eq}}\right) = -\frac{1}{R_{03}} \cdot \left(g_{m3} + \frac{1}{R_{03}} + \frac{1}{R_{eq}}\right) \]

plug into 1:

\[ I_{test} = \left(g_{m3} + \frac{1}{R_{03}}\right)(g_{m3} + \frac{1}{R_{03}}) + \frac{1}{R_{03}} \]
8. Show that the input resistance (R seen at V3 node) for the Wilson MOS mirror shown at right is given by \( 2/g_m \). Assume that all three transistors are identical and neglect the Early effect. (Hint: Use a test source and find the Thevenin equivalent resistance)

\[
I_{\text{test}} = g_m V_{g2} \\
V_{g2} = V_{g3} - V_{g1} = 0 \\
\text{Assume Equal transistors so } V_{g2} = V_{g1} = V_{g3} \\
2V_{g2} = 1 \\
V_{g2} = \frac{1}{2} \\
I_{\text{test}} = g_m \left( \frac{1}{2} \right) \\
R_{\text{in}} = \frac{1}{I_{\text{test}}} = \frac{2}{g_m}
\]
9. (a) For the circuit below, assume BJTs with high $\beta$ and $v_{BE}=0.8\text{V}$ at 1mA. Find the value of $R$ that will result in $I_o=10\mu\text{A}$.

(b) For the design in (a), find $R_o$ assuming $\beta=100$ and $V_A=50\text{V}$.
Assume diode-connected transistors are negligible.

\[ R_{out} = V_{test} = 1V \]

\[ I_{test} = \frac{1}{R} \]

\[ R = 68.5 \, k\Omega \]

\[ R_T = 250k \, \Omega \]

\[ g_m = 0.4m \]

Using node-\( V_1 \): 
1. \[ g_m V_T + \frac{(1 - (-V_T))}{R_o} - I_{test} = 0 \]
2. \[ \frac{-V_T}{R_T} + \frac{-V_T}{R} + \frac{(-V_T - 1)}{R_o} - g_m V_T = 0 \]

Combining terms in (2):

\[ V_T \left( \frac{1}{R_T} + \frac{1}{R} + \frac{1}{R_o} + g_m \right) = \frac{-1}{R_o} \]

where \( g_m = \frac{\beta}{R_T} \)

\[ V_T = \frac{-1}{R_o} = -\frac{1}{R_o} \cdot \left( \frac{R_T}{R_T + R + R_o} \right) = \frac{-\frac{5V}{101}}{250k + 68.5k + 5k} = -1.7 \, \text{V}^2 \]

Combining terms in (1):

\[ V_T \left( g_m + \frac{1}{R_o} \right) + \frac{1}{R} = I_{test} \]

\[ I_{test} = -4.78 \times 10^{-4} \left( \frac{1}{5m} + \frac{1}{5m} \right) + \frac{1}{5m} \approx 8.7 \, mA \]

\[ R_{out} = \frac{V_{test}}{I_{test}} = \frac{1}{8.7mA} = 114.9 \, \text{M} \Omega \]
10. If the pnp transistor in the circuit below is characterized by its exponential relationship with a scale current $I_S$, show that the dc current $I$ is determined by $IR = V_T\ln(I/I_S)$. Assume $Q_1$ and $Q_2$ to be matched and $Q_3$, $Q_4$, and $Q_5$ to be matched. Find the value of $R$ that yields a current $I = 100\,\text{A}$. For the BJT, $V_{EB} = 0.7\,\text{V}$ at $I_E = 1\,\text{mA}$.

\[ I = \frac{V_T}{R} \ln \left( \frac{I}{I_S} \right) \]

and $R$ can be determined as

\[ R = \frac{V_T}{I_S} \ln \left( \frac{I}{I_S} \right) \]

To calculate $R$, we find $I_S$:

\[ 1\,\text{mA} = I_S e^{v_{BE}/V_T} \text{ or} \]

\[ I_S = \frac{1\,\text{mA}}{e^{(0.7\,\text{V}/0.025\,\text{V})}} = 6.9 \times 10^{-13}\,\text{mA} \]

\[ R = \frac{V_T}{I_S} \ln \left( \frac{I}{I_S} \right) \]

\[ = \frac{(0.025\,\text{V}) \ln \left( \frac{0.1\,\text{mA}}{6.9 \times 10^{-13}\,\text{mA}} \right)}{0.1\,\text{mA}} = 6.42\,\text{kΩ} \]