Example

A \& B \text{ think of an "AND" gate}

\[ \begin{array}{c|c|c}
A & \& & B \\
\hline
2, 6 & \text{ON} & \text{ON} & \text{ON} \\
1, 5 & \text{ON} & \text{OFF} & \text{OFF} \\
4 & \text{OFF} & \text{ON} & \text{OFF} \\
3 & \text{OFF} & \text{OFF} & \text{OFF} \\
\end{array} \]

\[ \text{So the input } A \text{ to the AND gate is on if the outcome is in the list of outcomes in the definition of } A. \text{ Similarly for } B. \]

\[ \begin{array}{c|c|c|c}
\text{outcome} & A & B & A \& B \\
\hline
2, 6 & \text{ON} & \text{ON} & \text{ON} \\
1, 5 & \text{ON} & \text{OFF} & \text{OFF} \\
4 & \text{OFF} & \text{ON} & \text{OFF} \\
3 & \text{OFF} & \text{OFF} & \text{OFF} \\
\end{array} \]

\[ \text{This is the line that constitutes the list of outcomes in } A \& B, \text{ so } A \& B = \{2, 6\}. \]

A \lor B \text{ think of an "OR" gate}

\[ \begin{array}{c|c|c|c}
A & \lor & B \\
\hline
2, 6 & \text{ON} & \text{ON} & \text{ON} \\
1, 5 & \text{ON} & \text{OFF} & \text{ON} \\
4 & \text{OFF} & \text{ON} & \text{ON} \\
3 & \text{OFF} & \text{OFF} & \text{OFF} \\
\end{array} \]

\[ \text{These lines all constitute the list of outcomes in } A \lor B, \text{ so } A \lor B = \{2, 6, 1, 5, 4\}. \]
Think of a **NOT gate**

A → A'

<table>
<thead>
<tr>
<th>outcome</th>
<th>A</th>
<th>A'</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2, 5, 6</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>3, 4</td>
<td>off</td>
<td>on</td>
</tr>
</tbody>
</table>

Note: \((A')' = A\)

This is the line that lists the outcomes in \(A'\)

\(A' = \{3, 4\}\)

Notice that \(A \cap B = B \cap A\) and \(A \cup B = B \cup A\)

Define a third event \(C = \{3, 6\}\)

\[A \cap B \cap C = (A \cap B) \cap C = A \cap (B \cap C) = (A \cap C) \cap B\]

\[A \cup B \cup C = (A \cup B) \cup C = A \cup (B \cup C) = (A \cup C) \cup B\]

<table>
<thead>
<tr>
<th>outcomes</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>(A \cap B \cap C)</th>
<th>(A \cup B \cup C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>on</td>
<td>on</td>
<td>on</td>
<td>on</td>
<td>on</td>
</tr>
<tr>
<td>2</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>none</td>
<td>on</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>1, 5</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>none</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>4</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>3</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>off</td>
<td>on</td>
</tr>
<tr>
<td>none</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>off</td>
<td>off</td>
</tr>
</tbody>
</table>

\(\{63, 62, 15, 4, 3\}\)
De Morgan's laws:

\[ (A \cap B)' = (A' \cup B') \]

\[ (A \cup B)' = A' \cap B' \]

Distributivity:

\[ (A \cup B) \cap C = (A \cap C) \cup (B \cap C) \]

\[ (A \cap B) \cup C = (A \cup C) \cap (B \cup C) \]

Other properties:

\[ A \cup (A \cap B) = A \]

\[ A \cap (A \cup B) = A \]

\[ A \cap A' = \emptyset, \quad A \cup A' = S' \]

\[ A \cap \emptyset = \emptyset, \quad A \cup \emptyset = A \]