1. A continuous random variable $X$ has the probability density function

$$f(x) = \begin{cases} 
0, & x < 1 \\
hx - h, & 1 \leq x \leq 2 \\
3h - hx, & 2 \leq x \leq 3 \\
0, & x > 3 
\end{cases}$$

which can be graphed as

(a) Find $h$ which makes $f(x)$ a valid probability density function.
(b) Find the cumulative distribution function $F(x)$. 
2. Random variable $X$ has a normal probability distribution with mean 10.3 and standard deviation 2.

(a) Compute the numerical value of $P(7.2 \leq X \leq 13.8)$.

(b) Find a value $d$ such that $X$ is in the range $10.3 \pm d$ with probability 0.999.

(c) Let $Y$ be a random variable with variance $\sigma_Y^2 = 6$ and independent of $X$. Compute the variance of $5X - 3Y$. 
3. Let $X$ and $Y$ be two continuous random variables with the joint density function

$$f(x, y) = \begin{cases} 
  x + y, & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1 \\
  0, & \text{elsewhere}
\end{cases}$$

(a) Are the random variables $X$ and $Y$ independent? Justify your answer.
(b) Compute the numerical value of $P(Y \geq \frac{1}{2}, X \leq \frac{1}{2})$. 

4. Let $X$ be the sent bit and $Y$ the received bit in a binary communications channel. The joint probability distribution $f(x, y)$ is given as:

<table>
<thead>
<tr>
<th>$f(x, y)$</th>
<th>$x=0$</th>
<th>$x=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y=0$</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
<td>$y=1$</td>
<td>0.1</td>
<td>0.4</td>
</tr>
</tbody>
</table>

(a) Compute the numerical value of $P(Y = 1 | X = 0)$

(b) Compute the covariance of random variables $X$, $Y$.

(c) When a single bit is sent and received, we say that an error has occurred if $Y \neq X$. If a 8-bit long message is sent over this communication channel, what is the probability that 1 or less errors will occur?