1. Answer the following multiple choice questions by circling the correct answer:

(a) Which of the following confidence levels will result in a **narrower** confidence interval for the same sample size?
   (a) 99% (x) 95%

(b) Which of the following sample sizes will result in the **widest** confidence interval for the same confidence level?
   (a) n=100 (b) n=1000 (c) n=200 (d) All n give the same size interval

(c) If a hypothesis is rejected at significance level $\alpha = 0.001$, is it possible that the hypothesis is not rejected if the test was done at significance level $\alpha = 0.01$ (with everything else staying the same)?
   (a) Yes (x) No

(d) An experiment consists of taking the average of 100 rolls of a fair dice. Let $\bar{X}$ be the average of the 100 rolls. Does $\bar{X}$ have a normal distribution?
   (a) Yes (x) No

(e) A linear model is fitted to two different samples using least squares regression and the coefficient of determination ($R^2$) is computed for each sample. The first sample yields $R^2 = 0.37$ while the second sample yields $R^2 = 0.81$. For which sample is the linear model a better fit?
   (a) First sample (b) Second sample
2. An engineer wants to investigate the relationship between the electrical resistivity of a certain metal and temperature. Let $x$ be the temperature in degrees Fahrenheit. Let $Y$ be the electrical resistivity in $\text{n}\Omega \text{m}$. The engineer collects the following data:

<table>
<thead>
<tr>
<th>$i$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_i$</td>
<td>50</td>
<td>150</td>
<td>250</td>
</tr>
<tr>
<td>$y_i$</td>
<td>100</td>
<td>115</td>
<td>130</td>
</tr>
</tbody>
</table>

(a) Find the equation of the fitted regression line.
(b) Estimate the electrical resistivity of this metal at a temperature of 300 degrees Fahrenheit.

\[
\bar{x} = \frac{50 + 150 + 250}{3} = 150
\]
\[
\bar{y} = \frac{100 + 115 + 130}{3} = 115
\]
\[
b = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{200}{2000} = 0.1
\]
\[
a = \bar{y} - b\bar{x} = 115 - 0.1 \times 150 = 92.5
\]
\[
\hat{y} = 0.1x + 92.5
\]
\[
\hat{y} = \frac{3}{20} \times 300 + 92.5 = 137.5
\]

To estimate the electrical resistivity of this metal at a temperature of 300 degrees Fahrenheit, substitute $x = 300$: 
\[
\hat{y} = \frac{3}{20} \times 300 + 92.5 = 137.5
\]
3. A DC power supply manufacturer wants to test the hypothesis that the mean output voltage is 100 V (for a variety of different loads). Assume that the output voltage has a normal probability distribution.

A quality control engineer measures the output voltage when the supply is connected to 16 different loads and computes a sample mean \( \bar{x} = 103.6 \) and sample variance \( s^2 = 36 \).

(a) Perform a hypothesis test at significance level \( \alpha = 0.05 \) for the null hypothesis that the mean output voltage is 100 V.

(b) Find a 99% confidence interval for the population variance \( \sigma^2 \).

\[ H_0 : \mu = 100 \quad H_1 : \mu \neq 100 \]

Reject \( H_0 \) if

\[ \bar{x} > \mu_0 + t_{\alpha/2} \frac{s}{\sqrt{n}} \]

or \[ \bar{x} < \mu_0 - t_{\alpha/2} \frac{s}{\sqrt{n}} \]

\[ \bar{x} > 100 + 2.131 \frac{6}{\sqrt{16}} = 103.1965 \]

\[ \bar{x} < 100 - 2.131 \frac{6}{\sqrt{16}} = 96.8035 \]

\[ \bar{x} = 103.6 \quad \text{Reject } H_0 \]

\[ (n-1)s^2 < \chi^2 < \frac{(n-1)s^2}{\chi_{1-\alpha/2}^2} \]

\[ v = 15 \quad \chi^2_{0.025} = 32.801 \]

\[ \chi^2_{0.975} = 4.601 \]

\[ 15 \times 36 < \sigma^2 < 15 \times 36 \]

\[ \frac{36}{32.801} < \sigma^2 < \frac{15 \times 36}{4.601} \]
4) a)  if known \( n > 30 \) \( \Rightarrow \) z-distribution

\[ H_0: \mu = 100 \quad \alpha = 0.05 \]

\[ H_1: \mu \neq 100 \quad \mu_0 = 100 \]

Reject \( H_0 \) if \( \bar{x} < \mu_0 - \frac{z_{0.12} \cdot \sigma}{\sqrt{n}} \)

or \( \bar{x} > \mu_0 + \frac{z_{0.12} \cdot \sigma}{\sqrt{n}} \)

\[ z_{0.12} = z_{0.025} = 1.96 \]

Reject if \( \bar{x} < 100 - 1.96 \frac{6}{\sqrt{36}} = 98.04 \)

or \( \bar{x} > 100 + 1.96 \frac{6}{\sqrt{36}} = 101.96 \)

\( \bar{x} = 102 > 101.96 \) \( \Rightarrow \) **Reject**

b) From linear combination we know

that \( \mu_{x+y} = \mu_x + \mu_y = 100 + 100 = 200 \)

also \( \sigma^2_{x+y} = \sigma^2_x + \sigma^2_y = 6^2 + 6^2 = 72 \)

so \( \sigma_{x+y} = \sqrt{72} = 8.49 \)

\( \chi = 0.02 \) \( z \)-distribution \( z_{0.12} = z_{0.01} \approx 2.33 \)
\[ w - 2 \times \frac{3.0}{\sqrt{n}} < \mu_w < w + 2 \times \frac{3.0}{\sqrt{n}} \]

\[ 201 - 2.33 \times \frac{8.49}{\sqrt{100}} < \mu_w < 201 + 2.33 \times \frac{8.49}{\sqrt{100}} \]

198.022 < \mu_w < 202.978