One Port Negative-Resistance Oscillators

\[ Z_{in}(A, \omega) = R_{in}(A, \omega) + j X_{in}(A, \omega) \]

A \rightarrow \text{amplitude of } u(t)

\[ R_{in}(A, \omega) < 0 \]

The oscillator is constructed by connecting the device to a passive load impedance

\[ Z_{L}(\omega) = R_{L}(\omega) + j X_{L}(\omega) \]

The one-port Nlwo is stable if:

\[ \text{Re}\{Z_{in}(A, \omega) + Z_{L}(\omega)\} > 0 \]

For oscillation to occur, the loop gain must be unity

\[ \Gamma_{in}(j\omega) \Gamma_{L}(j\omega) = 1 \]
At the amplitude $A = A_0$ & frequency $\omega = \omega_0$, the Nlw will oscillate when

$$\Gamma_{\text{in}}(A_0, \omega_0) P_L(\omega_0) = 1$$  \hspace{1cm} -(1)

$$\Gamma_{\text{in}}(A_0, \omega_0) = \frac{Z_{\text{in}}(A_0, \omega_0) - Z_0}{Z_{\text{in}}(A_0, \omega_0) + Z_0}$$  \hspace{1cm} -(2)

$$P_L(\omega_0) = \frac{Z_L(\omega_0) - Z_0}{Z_L(\omega_0) + Z_0}$$  \hspace{1cm} -(3)

From the three equations we get

$$Z_{\text{in}}(A_0, \omega_0) + Z_L(\omega_0) = 0$$

Equating real & imaginary part

$$R_{\text{in}}(A_0, \omega_0) + R_L(\omega_0) = 0$$

$$X_{\text{in}}(A_0, \omega_0) + X_L(\omega_0) = 0$$

The device is defined to be unstable over some frequency range $\omega_1 < \omega < \omega_2$ if $R_{\text{in}}(A, \omega) < 0$.

The one-port Nlw is unstable for some $\omega$ in the range if the net resistance of the network is negative i.e.
\[ R_{\text{IN}}(A, \omega) > R_L(\omega) \] - (A)

Under proper conditions, a growing sinusoidal current will flow through the circuit i.e., at the start of oscillations, when amplitude \( A \) is small \( (A) \), must be satisfied. This is expressed in the form

\[ |R_{\text{IN}}(0, \omega)| > R_L(\omega) \]

The oscillations will continue to build up as long as the loop resistance is negative.

The amplitude of the current must eventually reach a steady-state value \((\text{i.e., at } A=A_0 \text{ and } \omega=\omega_0)\), which occurs when the loop resistance is zero.

\( Z_{\text{IN}}(A, \omega) \) is amplitude and frequency dependent. It is necessary to find another condition to guarantee a stable oscillation.

If the frequency dependence of \( Z_{\text{IN}}(A, \omega) \) can be neglected for small variations around \( \omega_0 \), Kurokawa has shown that a stable oscillation is obtained when \( (16) \) and \( (17) \) are satisfied and the following condition is also satisfied.
\[
\frac{\partial R_{\text{in}}(A)}{\partial A} \bigg|_{A=A_0} \cdot \frac{\partial X_e(\omega)}{\partial \omega} \bigg|_{\omega=\omega_0} - \frac{\partial X_{\text{in}}}{\partial A} \bigg|_{A=A_0} \frac{\partial R_L(\omega)}{\partial \omega} \bigg|_{\omega=\omega_0} > 0
\]

In many cases,
\[
\frac{\partial R_L(\omega)}{\partial \omega} < 0
\]

In a given oscillator design, the input impedance of the active device is known for small-signal conditions.

A practical way of designing \( R_L \) is to select the value of \( R_L \) for maximum oscillator power.

If magnitude of negative resistance is a linearly decreasing function of \( A \), we can express \( R_{\text{in}}(A) \) in the form
\[
R_{\text{in}}(A) = R_0 \left[ 1 - \frac{A}{A_m} \right]
\]

\(-R_0\) is value of \( R_{\text{in}}(A) \) at \( A=0 \) & \( A_m \) is maximum value of \( A \).

The power delivered to \( R_L \) by \( R_{\text{in}} \) (for \( A < A_m \))
\[
P = \frac{1}{2} \Re \{ V I^* \} = \frac{1}{2} I^2 R_{\text{in}}(A) > \frac{1}{2} A^2 R_0 \left[ 1 - \frac{A}{A_m} \right]
\]
Value of $A$ that maximizes the oscillation power is found from

$$\frac{dP}{dA} = \frac{1}{2} R_0 \left[ 2A - \frac{3A^2}{A_m} \right] = 0$$

which gives desired value of $A$, denoted by $A_{0,\text{max}}$ that maximizes the power

$$A_{0,\text{max}} = \frac{2}{3} A_m$$

At $A_{0,\text{max}}$ the value of $R_{\text{in}} (A_{0,\text{max}})$ is,

$$R_{\text{in}} (A_{0,\text{max}}) = -\frac{R_0}{3}$$

A convenient value of $R_L$, which maximizes the oscillator power

$$R_L = \frac{R_0}{3}$$
Two-port Negative-Resistance Oscillators.

- The general block diagrams for two-ports negative resistance oscillators are shown in Figure (a) & (b).

![Diagram of a two-port negative resistance oscillator](image)

- Either port can be used as the terminating port.
- Once terminating port is selected, the other port is referred to as the input port.
- The load-matching N/ω is connected to the input port in accordance with the one port notation.
- When the two-port is potentially unstable, an appropriate \( Z_T \) permits the two-port to be represented as a one-port negative resistance device with input impedance \( Z_W \). The conditions for a stable oscillation are:
\[ R_{\text{in}}(A_0, \omega_0) + R_L(\omega_0) = 0 \]
\[ X_{\text{in}}(A_0, \omega_0) + X_L(\omega_0) = 0 \]

\[ \left| \frac{\partial R_{\text{in}}(A)}{\partial A} \bigg|_{A=A_0} \frac{\partial X_L(\omega)}{\partial \omega} \bigg|_{\omega=\omega_0} - \frac{\partial X_{\text{in}}(A)}{\partial A} \bigg|_{A=A_0} \frac{\partial R_L(\omega)}{\partial \omega} \bigg|_{\omega=\omega_0} \right| > 0 \]

→ To start the oscillation, the value of \( R_L \) is selected according to

\[ R_L = R_0 \]

For general \( R_L = \left| \frac{R_{\text{in}}(0, \omega)}{3} \right| \)

When \( ilp \) port is made to oscillate, the terminal \( ilp \) port also oscillated. The fact that both ports are oscillating can be proved as follows.

→ \( ilp \) port is oscillating when

\[ \Gamma_{\text{in}} \Gamma_L = 1 \]

\[ \Gamma_L = \frac{1}{\Gamma_{\text{in}}} = \frac{1-S_{22} \Gamma_T}{S_{11} - \Delta \Gamma_T} \]

\[ \Gamma_T = \frac{1-S_{11} \Gamma_L}{S_{22} - \Delta \Gamma_L} \]
\[ \Gamma_{\text{out}} = \frac{S_{22} - \Delta \Gamma_2}{1 - S_{11} \Gamma_2} \]

\[ \Gamma_{\text{out}} \Gamma_T = 1 \]

which shows that the terminating port is also oscillating.

A design procedure for a two-port oscillator is as follows.

1) Use a potentially unstable transistor at the frequency of oscillation \( \omega_0 \).

2) Design the terminating network to make \( |P_{in}| > 1 \).

3) Design the load network to resonate \( Z_{in} \), and to satisfy the start of oscillation condition, i.e.

\[ X_L(\omega_0) = -X_{in}(\omega_0) \]

\[ R_e = \frac{R_0}{3} \]
5.3.1 Design 8 G Hz GaAs FET

\[ S_{11} = 0.98 \angle 63^\circ \quad S_{12} = 0.39 \angle -54^\circ \]
\[ S_{21} = 0.675 \angle 161^\circ \quad S_{22} = 0.465 \angle 120^\circ \]

SoIn

1. \( K = \frac{1}{2} \quad K = 0.529 \)

2. Draw Terminating stability O
   a) Calculate
   \[ Z_S = \left| \frac{S_{12} S_{21}}{1 - |S_{11}|^2 - |\Delta|} \right| \quad C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2} \]

3. Choose a point in the unstable region
   in the example we choose point A. \( \Gamma_T = 1 \angle -163^\circ \)
   The associated impedance \( Z_T = -j7.5 \Omega \)

4. With \( Z_T \) connected the input reflection coefficient is found:
   \[ \Gamma_{IN} = \frac{S_{11} - \Delta \Gamma_T}{1 - S_{22} \Gamma_T} \quad \Gamma_{IN}^2 S_{11} + \frac{S_{12} S_{21} \Gamma_T}{1 - S_{22} \Gamma_T} \]

5. Calculate \( Z_{IN} = -58 - j2.6 \Omega \)
Load matching \( N / \omega \) can be modeled as:
\[
X_L(\omega_0) = -X_{IN}(\omega_0)
\]
\[
R_L = \frac{R_0}{3}
\]
\[
Z_L = 19 + j 2.6 \omega
\]

Example 2

\[
S_{11} = 0.5 \angle -95^\circ
\]
\[
S_{21} = 1.1 \angle 44^\circ
\]
\[
S_{12} = 0.35 \angle -31^\circ
\]
\[
S_{22} = 0.8 \angle 46^\circ
\]
\[
\mathbf{S} = 9 \text{GHz}
\]

So,

\[
\text{Calculate } K \text{ & } \Lambda_1
\]

\[
K = 0.355 \quad \Lambda_1 = 0.405
\]
Since $K<1$ the transistor is potentially unstable.

(2) We use port 2 as the terminating port.

(3) Calculate $g_t$ & $C_t$

\[
g_t = \frac{g_{12} S_{21}}{|S_{22}|^2 - |\Delta|^2}
\]

\[
C_t = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}
\]

We get $g_t$ & $C_t$

$g_t = 0.808 \quad C_t = 1.493 \angle 59.8^\circ$

(4) Any $\Gamma_t$ in the shaded region will produce a negative resistance.

The reflection coefficient $\Gamma_t$ is selected at a point for eq (1 $\angle 67.38^\circ$)

$Z_t = -j 75 \angle 90^\circ$

(5) \[
\Gamma_{IN} = \frac{S_{11} + S_{12} S_{21} \Gamma_t}{1 - S_{22} \Gamma_t}
\]

$\Gamma_{IN} = 1.49 \angle 100.48^\circ$

$Z_{IN} = 16.25 - j 38.95$
\[ X_L - X_{in} = 38.95 \Omega \]

\[ R_V = \frac{1|R_{in}|}{3} = 16.25 \frac{\Omega}{3} = 5.42 \Omega \]

For negative resistance oscillators at microwave frequencies using BJTs, the CB (Common base) configuration is normally used. For FETs, the CG configuration is commonly used.

In these configurations, the transistors are usually potentially unstable, and this is a very desirable condition in oscillator design.

- In negative resistance oscillators, the capacitances of the transistor provide some or all of the feedback needed for oscillation.

- A properly designed series feedback network can significantly increase the negative resistance presented by the two-port network.

The unstable region of the two-port network can be enhanced by the use of series feedback.
For BJTs a series feedback inductor of a few nanohenries enhances the unstable region. For FETs either a series-feedback inductor or a series-feedback capacitor of a few picofarads enhances the unstable region.

The purpose of a series-feedback element is to provide positive feedback making the configuration more unstable.

Method for designing Series feedback Nl/W

\[
\begin{array}{c}
I_1 \\
\uparrow \\
+ \\
V_1 \\
- \\
\downarrow \\
\text{Transistor} \\
\uparrow \\
I_2 \\
\uparrow \\
+ \\
V_2 \\
- \\
\downarrow \\
\end{array}
\]

[2 port Nl/W]

Let \([S]\) be the 3-port scattering matrix & \([S]\) be the two-port scattering matrix

\[
\begin{array}{c}
I_1 \\
\uparrow \\
+ \\
V_1 \\
- \\
\downarrow \\
\text{Transistor} \\
\uparrow \\
I_2 \\
\uparrow \\
+ \\
V_2 \\
\downarrow \\
\end{array}
\]

\[
\begin{array}{c}
I_3 \\
\uparrow \\
+ \\
V_3 \\
- \\
\downarrow \\
\end{array}
\]
The relation between 2-port & 3-port S-parameters can be written as:

\[
\hat{S}_{11} = S_{11} + \frac{\sigma_{11} \sigma_{12}}{4 - \sigma}
\]

\[
\hat{S}_{21} = S_{21} + \frac{\sigma_{22} \sigma_{12}}{4 - \sigma}
\]

\[
\hat{S}_{12} = S_{12} + \frac{\sigma_{11} \sigma_{21}}{4 - \sigma}
\]

\[
\hat{S}_{13} = \frac{2 \sigma_{11}}{4 - \sigma}
\]

\[
\hat{S}_{23} = \frac{2 \sigma_{22}}{4 - \sigma}
\]

\[
\hat{S}_{33} = \frac{\sigma}{4 - \sigma}
\]

\[
S_{11} = 1 - S_{11} - S_{12}
\]

\[
\sigma_{11} = 1 - S_{11} - S_{21}
\]

\[
\sigma_{21} = 1 - S_{22} - S_{12}
\]

\[
\sigma_{22} = 1 - S_{22} - S_{21}
\]

\[
\sigma = S_{11} + S_{12} + S_{21} + S_{22}
\]

**\( S \)**

\[
S_{11} = \hat{S}_{11} - \frac{\hat{S}_{13} \hat{S}_{31}}{1 + \hat{S}_{33}}
\]

\[
S_{12} = \hat{S}_{12} - \frac{\hat{S}_{13} \hat{S}_{32}}{1 + \hat{S}_{33}}
\]

\[
S_{21} = \hat{S}_{21} - \frac{\hat{S}_{23} \hat{S}_{31}}{1 + \hat{S}_{33}}
\]

\[
S_{22} = \hat{S}_{22} - \frac{\hat{S}_{23} \hat{S}_{32}}{1 + \hat{S}_{33}}
\]
If a series feedback impedance is used, as shown in figure below

\[ Q_{3} \approx 1 / f b_{3} \]

1 + \( S_{33} \) is replaced by \( \frac{S_{33}}{1 + S_{33}} \)

\[ S_{11} = \frac{A_{1} \Gamma_f - S_{11}^{\hat{}}}{\hat{S}_{33}^{\hat{}} \Gamma_f - 1} \]

\[ S_{22} = \frac{A_{2} \Gamma_f - S_{22}^{\hat{}}}{\hat{S}_{33}^{\hat{}} \Gamma_f - 1} \]

\[ A_{1} = S_{11} \hat{S}_{33} - \hat{S}_{13} \hat{S}_{31} \]

\[ A_{2} = S_{22} \hat{S}_{33} - \hat{S}_{23} \hat{S}_{32} \]
The mapping of $|S_{ij}|$ onto the $S_{11}$ and $S_{22}$ planes have centers $\mathbf{e}$ and radii given by

$$C_1 = \hat{S}_{11} - \Delta_1 \hat{S}_{33}$$

$$\rho_1 = \frac{1 \hat{S}_{13} \hat{S}_{21}}{|1 - |\hat{S}_{33}|^2|}$$

and

$$C_2 = \hat{S}_{22} - \Delta_2 \hat{S}_{33}$$

$$\rho_2 = \frac{1 \hat{S}_{23} \hat{S}_{32}}{|1 - |\hat{S}_{33}|^2|}$$

for the $S_{11}$ and $S_{22}$ planes.

The maximum value of $|S_{11}|$ occurs at a point where

$$S_{11}(\text{max}) = (1C_1 + \rho_1') \mathbf{e}$$

$\rho_1'(\text{max})$ is given by
\[ \beta_e = j \chi_{i, \text{max}} = \frac{\Delta_1 + \hat{S}_{11} - S_{11, \text{max}}(1 + \hat{S}_{23})}{\Delta_1 - \hat{S}_{11} + S_{11, \text{max}}(1 - \hat{S}_{33})} \]

To properly orient the mapping, the \( \beta_e = \infty \) point maps onto the point

\[ \beta_{11} = \frac{\Delta_1 - \hat{S}_{11}}{\hat{S}_{33} - 1} \]

\( \beta_{33} = 0 \) maps onto the point

\[ \beta_{33} = \frac{\Delta_1 + \hat{S}_{11}}{\hat{S}_{33} + 1} \]

\[ \| \vec{y} \| \text{ we obtain} \]

\[ \beta_{22} = (1 \overline{c_2} \ell^2 + \overline{y_2} \ell) \ell \overline{c_2} \]

\[ \beta_{33} = j \chi_{2, \text{max}} = \frac{\Delta_2 + \hat{S}_{22} - S_{22, \text{max}}(1 + \hat{S}_{23})}{\Delta_2 - \hat{S}_{22} + S_{22, \text{max}}(1 - \hat{S}_{33})} \]

\( \beta_{33} = \infty \) point maps onto

\[ \beta_{22} = \frac{\Delta_2 - \hat{S}_{22}}{\hat{S}_{33} - 1} \]
and \( z_f = 0 \) point maps to

\[
S_{22} = \frac{A_z + \hat{S}_{22}}{\hat{S}_{33} + 1}
\]

Design Procedure

1. Convert 2 port S parameter to 3-port
2. Calculate \( C_i', r_i', C_2' \& r_2' \)
3. Calculate \( S_{11}(max), \hat{X}_1(max), S_{22}(max) \& \hat{X}_2(max) \)
4. Plot mapping of the \( T \) plane onto the \( S_{11} \& S_{22} \) planes.
5. Select appropriate \( z_f \) value.
Design a 2.75 GHz oscillator using a BJT in common-base configuration. The S-parameters at 2.75 GHz, at a given Q point are:

\[ S_{11} = 0.9 \angle 150^\circ \]
\[ S_{21} = 1.7 \angle -80^\circ \]
\[ S_{12} = 0.07 \angle 120^\circ \]
\[ S_{22} = 1.08 \angle -58^\circ \]

**Solution**

1) Calculate \( K \) (\( K = -0.64 \))

We can use feedback to increase the narrowband instability. With \( L = 1.45 \mu H \),

The new S-parameters are:

\[ S_{11} = 1.72 \angle 100^\circ \]
\[ S_{21} = 2.08 \angle -136^\circ \]
\[ S_{12} = 0.712 \angle 94^\circ \]
\[ S_{22} = 1.16 \angle -102^\circ \]
Either port can be used as the terminating port. The emitter-to-ground port was selected for this example for the load matching Nlw, and the collector to ground port for the terminating Nlw.

From the Smith chart we observe that the center of the Smith chart is unstable. Hence a 50Ω termination will ensure |Pw| > 1.

Although this is simple there are a couple of issues:
1) For associated values of Pw the required value of Pc for the oscillator might be difficult to implement.
2) Some tuning capabilities are needed in the matching Nlw to attain the desired frequency of oscillation.

The values of Pw and Pc are related by:

\[ P_w = S_{11} + \frac{S_{12} S_{21} P_c}{1 - S_{22} P_c} \]

One way is choosing Pc in the unstable region & calculating the associated value of Pw.
Oscillator Design Using Large-Signal Measurements

→ The previous design methods provide good practical results; however, there is no assurance that the oscillator power is optimum.

→ Here we develop a method that optimizes the oscillator power, based on large signal measurements.

→ The method consists of designing the termination N|W so that the two port presents a negative resistance at the i/p port.

→ The resulting one port negative resistance N|W can be placed in a nonoscillatory circuit and be optimum load impedance as a function of power can be measured.

→ The termination N|W is designed so that Z|W presents a positive impedance at the load port that has a real part with magnitude smaller than 50Ω.

→ This is necessary because the equipment used to measure the large signal characteristics of the circuit has a source impedance, and the total loop resistance must be positive in order to avoid oscillatory clump measurements.
The large signal characterization is achieved, by measuring in the circuit shown below, the current amplitude and the impedance \( Z_{\text{in}}(A_i, w_0) \), as \( V_s \) is varied. The measurements are made at the desired frequency of oscillation \( w_0 \), and the source resistance is typically 50 \( \Omega \).

\[
I(A_i, w_0) = \frac{V_s}{R_s + R_{\text{in}}(A_i, w_0) + jX_{\text{in}}(A_i, w_0)}
\]

\[
P_{\text{th}}(A_i, w_0) = \frac{1}{2} |I(A_i, w_0)|^2 |R_{\text{in}}(A_i, w_0)|
\]

Measurement of \( P_{\text{th}}(A_i, w_0) \) versus \( Z_{\text{in}}(A_i, w_0) \) generates the large signal characteristics of the one port.\( \omega \).
If the one port is now terminated in the load impedance
\[ Z_L(w_0) = Z_{in}(A_1w_0) \]

The power delivered to \( Z_L \) is given by
\[ P_L(A_1w_0) = P_{in}(A_1w_0) \]

Measuring \( I(A_1w_0) \) at microwave frequencies is difficult.

The reflection coefficient \( R_{in}(A_1w_0) \) as a function of the available power from the source is measured. The available power from the source is given by
\[ P_{avs} = \frac{V_s^2}{8R_s} \]

The power added \( P_{add} \) (i.e., reflected power available in power) is given by
\[ P_{add} = P_{avs}(1R_{in}^2-1) \]

\[ P_{add}(A_1w_0) = \frac{V_s^2 |R_{in}(A_1w_0)|}{2 [ (R_{in}(A_1w_0) + R_s)^2 + X_{in}^2 ]} \]

i.e.,
\[ P_{add}(A_1w_0) = \frac{1}{2} \left( V_s |I(A_1w_0)| \right)^2 |R_{in}(A_1w_0)| \]
which shows that the added power is the power that the one port N/w will deliver to load \( Z_l(\omega) = Z_{\text{in}}(\omega_0) \).

The design procedure uses the small-signal \( S \) parameters to establish the terminal impedance that results in a negative resistance one-port N/w. Then the one-port oscillator performance is described by the measured large-signal characteristics

\[
\Gamma_{\text{in}} = \frac{S_{11} - \Delta S_{11} \Gamma_T}{1 - S_{21} \Gamma_T}
\]

describes the mapping of \( \Gamma_T \) plane into \( \Gamma_{\text{in}} \) plane. The mapping of the \( \omega \) \( \Gamma_T \| \Gamma_T \) into the \( \Gamma_{\text{in}} \) plane gives information about the passive impedance at the terminating port (i.e. \( \Gamma_T < 1 \)) that will make \( |\Gamma_{\text{in}}| > 1 \).

\[
C_{\text{in}} = \frac{S_{11} - \Delta S_{11} \Gamma_T}{1 - S_{21} \Gamma_T^2}
\]
\[ r_{IN} = \frac{S_{12} S_{21}}{(1 - |S_{22}|^2)} \]

Largest (\(\Gamma_{\text{IN}}\)) is obtained when we select value of \(\Gamma_{\text{IN}}\) on \(|\Gamma_{\text{T}}| = 1\) that has the same phase as \(C_{\text{IN}}\).

The value of \(\Gamma_{\text{T}}\) associated with \(\Gamma_{\text{IN,\max}}\) produces the largest negative resistance for \(Z_{\text{IN}}\), which can be realized using a passive terminating impedance.

The magnitude of \(\Gamma_{\text{IN,\max}}\) at \(a\) is

\[ |\Gamma_{\text{IN}}| = |C_{\text{IN}}| + r_{\text{IN}} \]

phase

\[ \angle \Gamma_{\text{IN}} = \angle C_{\text{IN}} \]

\[ \Gamma_{\text{IN,\max}} = (|C_{\text{IN}}| + r_{\text{IN}}) |L_{\text{IN}}| \]

Value of \(\Gamma_{\text{T}}\) at point \(a\),

\[ \Gamma_{\text{T,0}} = \frac{\Gamma_{\text{IN,\max}} - S_{11}}{\Gamma_{\text{IN,\max}} S_{22} - \Delta} \]