\[ S_{li} = 0.73 \angle 75^\circ \] \[ S_{l2} = 0 \] \[ S_{l2} = 0.21 \angle 80^\circ \]

1) optimum terminations

\[ \Gamma_s = S_{li}^* = 0.73 \angle 75^\circ \]
\[ \Gamma_l = S_{l2}^* = 0.21 \angle 80^\circ \]

Plot these on Smith chart

From the Smith chart we have

\[ Z_s = 50 \angle 2^\circ \]
\[ Z_l = 50(0.97 + j0.5) \]
\[ Z_L = 50 \angle 2^\circ \]

b) Calculate \( G_{S_{\text{max}}} \), \( G_{L_{\text{max}}} \), and \( G_{T_{\max}} \)

\[ G_{S_{\text{max}}} = \frac{1}{1 - |S_{li}|^2} = 2.141 \text{ or } 3.31 \text{ dB} \]

\[ G_{L_{\text{max}}} = \frac{1}{1 - |S_{l2}|^2} = 1.046 \text{ or } 0.195 \text{ dB} \]

\[ G_0 = 18 \angle 2^\circ = 19.8 \text{ or } 12.97 \text{ dB} \]

\[ G_{T_{\max}} \text{ (dB)} = 3.31 + 12.97 + 0.195 = 16.47 \text{ dB} \]
The Complete Smith Chart
Black Magic Design

Matching
ckt design.

50() (.47)

j(0.1n)
c) Draw several \( G_s \) constant \( \Omega \) (See Smith chart)

a) Design a \( \lambda \) matching \( \lambda/\lambda \)

\[ \Gamma = 0.2 \angle 80^\circ \]

Any \( \Gamma \) along \( G_s = 2 \text{dB} \) provides constant gain. We choose point A & perform our matching.

Potentially Unstable Case \[ |S_{ii}| > 1 \]

| \( S_{ii} \) | > 1 and it is possible for a passive termination to produce an infinite value of \( G_i \). The infinite value of \( G_i \) is produced by the critical value of \( \Pi_i \), called \( \Pi_{ic} \), given by

\[ \Pi_{ic} = \frac{1}{S_{ii}} \]  

Equation (1) states that the real part of the impedance associated with \( \Pi_{ic} \) is equal to the magnitude of the negative resistance associated with \( S_{ii} \). Therefore the total \( \lambda \) & output loop resistance is zero, and oscillations will occur.
\( g_i \) defined as

\[
g_i = G_i \left( \frac{1 - |S_{ii}|^2}{1 - |S_{ii}|^2 C(1 - g_i)} \right)
\]

\( g_i \) can attain -ve values because \( |S_{ii}| < 1 \) the constant \( G_i \) is given by

\[
C_{g_i} = \frac{g_i S_{ii}^*}{1 - |S_{ii}|^2 C (1 - g_i)}
\]

and the radius is given by

\[
r_{g_i} = \sqrt{1 - g_i C(1 - |S_{ii}|^2)}
\]

Gain \( G_i \) is infinite at \( \Pi_i = \Pi_i \), i.e. \( \frac{1}{S_{ii}} \). Since argument of \( C_{g_i} \) (re \( S_{ii}^* \)) is identical to argument of \( \frac{1}{S_{ii}} \).

It follows that center of the \( \Omega \)s are located along a line drawn from the origin to the point \( \frac{1}{S_{ii}} \).

The -ve resistance can be calculated using the Smith chart by localizing the point \( \frac{1}{S_{ii}} \) and interpreting the resistance as a being negative and the reactance as labeled.
The Complete Smith Chart
Black Magic Design

Plotting gain (for potentially unstable systems)

3.4.2

a) \( |S_{11}| > 1 \)
ve resistive
plot \( \frac{S_{11}^*}{|S_{11}|^2} = 0.44 \angle 120^\circ \)

\( Z_i = 50 \ (0.6 + j 1.04) \)

b) \( g_s = \frac{\sqrt{1 - g_i} \ \ (1 - |S_{11}|^2)}{(-1|S_{11}|^2)(1 - g_i)} \)

\( I_{gs} = \frac{g_s S_{11}^*}{1 - |S_{11}|^2} (1 - g_i) \)

\( g_i = \frac{C(1 - |S_{11}|^2)}{1 - |S_{11}|^2} \)

\( P_i = 0.6 \angle 120^\circ \)

Plot this point (B)

\( Z_r = 50 \ (0.6 + j 1.04) \)
To prevent oscillations in the input output $\Pi$, $\Pi$ must be selected such that the real part of the terminations impedance is larger than the magnitude of the negative resistance associated with the point $1/S_{ii}$. When a negative resistance occurs at the input, the stable region is that region where values of $\Pi$ produce a source impedance such that

$$\text{Re}(Z_S) > |\text{Re}(Z_{in})|$$

$$\text{Re}(Z_L) > |\text{Re}(Z_{out})|$$

Unilateral Figure of Merit

What is the error involved for assuming $S_{12} = 0$

$$G_{11} = \frac{1}{G_{11} G_{12}} \frac{1}{(1 - X)^2}$$

$$X = \frac{S_{12} S_{21} \Gamma_S \Gamma_L}{(1 - S_{ii} \Gamma_S) (1 - S_{zz} \Gamma_L)}$$

$$G_{11} = \frac{1 - |\Gamma_S|^2}{1 - S_{ii} \Gamma_S}$$

$$\frac{1 - |\Gamma_S|^2}{1 - S_{ii} \Gamma_S}$$

$$G_{12} = \frac{1 - |\Gamma_L|^2}{1 - \Gamma_{out} \Gamma_L}$$

$$G_{12} = \frac{1 - |\Gamma_L|^2}{1 - \Gamma_{out} \Gamma_L}$$

$$G_{12} = \frac{1 - |\Gamma_L|^2}{1 - \Gamma_{out} \Gamma_L}$$
The ratio of transducer power gain to the unilateral transducer power gain is bounded by:

\[
\frac{1}{(1+|x|)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-|x|)^2}
\]

When \( T = S_{11}^* \) and \( P = S_{22}^* \), \( G_{TU} \) has a maximum value and, in this case, the maximum error introduced when using \( G_{TU} \) is bounded by

\[
\frac{1}{(1+U)^2} < \frac{G_T}{G_{TU}} < \frac{1}{(1-U)^2}
\]

\[
U = \frac{|S_{12}| |S_{21}| |S_{11}| |S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}
\]

is the unilateral figure of merit.

\( U \) varies with frequency.
Simultaneous Conjugate match (Bilateral case)

$s_{12} \neq 0$ unilateral assumption cannot be made.

The input and output reflection coefficients are given by

$$\Gamma_{IN} = S_{11} + \frac{s_{12} s_{21} \Gamma_L}{1 - s_{22} \Gamma_L} - (1)$$

$$\Gamma_{OUT} = S_{22} + \frac{s_{12} s_{21} \Gamma_S}{1 - s_{11} \Gamma_S} - (2)$$

The condition required to obtain maximum transducer power gain are:

$$\Gamma_S^* \geq \Gamma_{IN}^* - (3)$$

$$\Gamma_L^* \leq \Gamma_{OUT}^* - (4)$$

From (1), (2), (3), and (4) we get

$$\Gamma_S^* = S_{11} + \frac{s_{12} s_{21} \Gamma_L}{1 - s_{22} \Gamma_L} - (5)$$

$$\Gamma_L^* = S_{22} + \frac{s_{12} s_{21} \Gamma_S}{1 - s_{11} \Gamma_S} - (6)$$
From (5) and (6) we get

\[ \Gamma_s = S_{11} + \frac{S_{12}S_{21}^*}{(1/r_{11}^*) - S_{22}^*} \quad - (7) \]

\[ \Gamma_L = \frac{S_{22} - (S_{11}S_{22} - S_{12}S_{21}) r_s}{1 - S_{11} r_s} = \frac{S_{22} - \Gamma_s r_s}{1 - S_{11} r_s} \quad - (8) \]

Solving (8) in (7) we get

\[ \Gamma_s = S_{11} + \frac{S_{12}S_{21}^*}{\left[ (1 - S_{11} r_s) / (S_{22} - \Gamma_s r_s) \right] - S_{22}^*} \]

Solving this we get \( \Gamma_{ms} \) and \( \Gamma_{ml} \), these conjugate match are obtained as:

\[ \Gamma_{ms} = B_1 \pm \frac{\sqrt{B_1^2 - 4 |C_1|^2}}{2 C_1} \quad - (9) \]

\[ \Gamma_{ml} = B_2 \pm \frac{\sqrt{B_2^2 - 4 |C_2|^2}}{2 C_2} \quad - (10) \]

\[ B_1 = 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \]

\[ B_2 = 1 + |S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \]

\[ C_1 = S_{11} - \Delta S_{22}^* \quad \text{and} \quad C_2 = S_{22} - \Delta S_{11}^* \]
if $|B_i| 2c_i > 1$ and $B_i > 0$ then $|\eta_{ms}| < 1$ when we use the sign in eqn (9) and $|\eta_{ms}| > 1$ when we use the sign in eqn (9)

if $|B_i| 2c_i > 1$ and $B_i < 0$ then $|\eta_{ms}| > 1$ when we use the sign in eqn (9) and $|\eta_{ms}| < 1$ when we use the sign in eqn (9)

**Observation:** $|B_i| 2c_i > 1$ implies that $|K| > 1$

**Proof:**

$$\left| \frac{B_i}{2c_i} \right| > 1 \Rightarrow \left| \frac{1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2}{2 (S_{11} - S_{22} \Delta)} \right| > 1$$

or

$$\left| 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \right| > 2 \left| (S_{11} - S_{22} \Delta) \right|$$

Squaring both sides

$$\left| 1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \right|^2 > 4 \left| (S_{11} - S_{22} \Delta) \right|^2$$

and

$$|S_{11} - S_{22} \Delta|^2 = (S_{11} - S_{22} \Delta) (S_{11}^* - S_{22} \Delta^*)$$

$$= |S_{12} S_{21}|^2 + (1 - |S_{22}|^2) \left( |S_{11}|^2 - |\Delta|^2 \right)$$
Therefore

\[ |1+ S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 |^2 > 4 |S_{12} S_{21}|^2 + 4 (1 - |S_{22}|^2) (|S_{11}|^2 - |\Delta|^2) > 4 |S_{12} S_{21}|^2 \]

or

\[ |1 + |S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 |^2 - 4 (1 - |S_{22}|^2) (|S_{11}|^2 - |\Delta|^2) > 4 |S_{12} S_{21}|^2 \]

or

\[ (1 - |S_{22}|^2 - |S_{11}|^2 + |\Delta|^2)^2 > 4 |S_{12} S_{21}|^2 \]

or

\[ 1 - |S_{22}|^2 - |S_{11}|^2 + |\Delta|^2 > 2 |S_{12} S_{21}| \]

\[ \therefore \quad \frac{|1 - |S_{22}|^2 - |S_{11}|^2 + |\Delta|^2|}{2 |S_{12} S_{21}|} > 1 \Rightarrow k > 1 \]

If

\[ |k| > 1 \quad \text{and} \quad \text{with } k \text{ positive, one solution of } \Gamma_M \text{ and } \Gamma_{ML} \text{ has a magnitude less than } 1, \text{ and } \] the other solution has a magnitude greater than 1.

For \( k > 1 \) and \( B_i > 0 \), the solutions with the minus sign have magnitude less than 1.
Also $|\Delta| < 1$ implies that $B_1 > 0$ and $B_2 > 0$. We have to use the equation for $\Pi_{ms}$ and $\Pi_{ml}$

The maximum transducer power gain, under simultaneous conjugate match is obtained from $G_T$ with $\Pi_s = \Pi_{in} = \Pi_{ms}$ and $\Pi_l = \Pi_{out} = \Pi_{ml}$

$$G_{T, \text{max}} = \frac{1}{1 - |\Pi_{ms}|^2} \frac{|S_{21}|^2}{|1 - |\Pi_{ml}|^2|} (1)$$

Substituting $\Pi_{ms}, \Pi_{ml}$ and into $(1)$, it can be shown that $G_{T, \text{max}}$ can be expressed in the form

$$G_{T, \text{max}} = \frac{|S_{21}|}{|S_{12}|} (K - \sqrt{K^2 - 1})$$

Since under simultaneous conjugate match condition $G_T = G_P = G_A$, it follows that $G_{T, \text{max}} = G_{P, \text{max}} = G_{A, \text{max}}$.

The maximum stable gain is defined as the value
d) $G_{T, \text{max}}$ when $K = 1$

$$G_{T, \text{max}} = \frac{|S_{21}|}{|S_{12}|}$$
For a potentially unstable transistor, $G_{ms}$ is the figure of merit.

A simultaneous conjugate match does not exist for $K < 1$. In a potentially unstable two-port network with $K > 1$ but $|\alpha| > 1$ solutions to $\Gamma_m$ and $\Gamma_m^*$ using the plus sign produce $|\Gamma_{ms}| < 1$ and $|\Gamma_{ml}| < 1$. In such a case the values of $\Gamma_{ms}$ and $\Gamma_{ml}$ given by using the two sign results is a minimum value of $G_r$, and if the VSWR are unity. Substituting these values of $\Gamma_{ms}$ and $\Gamma_{ml}$ into equation of $G_{r,\text{min}}$, the minimum value of $G_r$ is given by

$$G_{r,\text{min}} = \frac{|S_{21}|}{|S_{12}|} \left( K + \sqrt{K^2 - 1} \right)$$