CNF-SAT: a search procedure...

- Find assignment to variables s.t. each clause is true
- From inputs to outputs, build clauses and conjunct them

\[(Z + \lambda)(Z + X)(\lambda + X + Z)\]

- CNF
- Boolean formula: How do we build clauses from Gates?

\[(Z \leftarrow (\lambda + X))(\lambda + X \leftarrow Z) \Leftarrow \lambda + X = Z\]

Clause Normal Form for Boolean Satisfiability
CRASP: Intelligent decision making + conflict resolution

• 3-SAT is intractable. Heuristics to guide the search

  (multiplexers)

Hard SAT instance: very few solutions (e.g., prime numbers for

Infeasible/empty SAT instance: no point in the solution space

Solutions

Easy SAT instance: search space well distributed with many

Notice conflicting requirements of SAT engines

If no solution exists, prove it, and prove it quickly

Motivations for an Efficient Search
What if no solution exists? Explore the whole tree...

Select \( p = 1 \) and deduce \( c = 0 \)

\[ \vdash \text{Conflict: current assignment no-good. Backtrack} \]

Select \( q = 0 \)

\[ \vdash \text{Solve} \ f : \text{Select} \ a = 0 \]

Conflicts will lead you to backtrack chronologically

Will generate a Shannon tree

Select an assignment, simplify clauses & proceed...

\[ (r + q)(c + q)(c + p)(r + q + a) = f \]

The Search Process: A Decision Tree
Proceeding such... Decision Level 6: Select $x^1 = I$

$$(8x + 4x)(8x + 1x)(2x + lx)(9x + \frac{9}{2}x)\left(\frac{9}{2}x + \frac{9}{4}x\right) + \frac{9}{4}x + \frac{9}{8}x + \frac{9}{16}x + \frac{9}{3}x + \frac{9}{5}x + \frac{9}{7}x + \frac{9}{9}x)(3x + 3x)(2x + lx) = f$$

$0 = 11x = x^0 x^1 = I$. Decision Level 3: $x^2 = x^1 x^3 = I$. Decision Level 2: $x^3 = x^2 x^0 = I$. Decision Level 1: Effective assignment $x^0 = 0$

$$(3x + 8x + 4x + 2x + 1x)(2x + 1x + 1x + 1x + 1x + 1x + 1x + 1x + 1x + 1x) = f$$

**Conflict Analysis via Implication Graph**
\[(\Pi x + 0\Pi x + 6x + 1\Pi x) \text{ Converse:} \quad 0 = 1\Pi x; 0 = 0\Pi x; 0 = 6x; 1 = x \]

What caused the conflict?

Implication Graph for Conflict Analysis
\[(\delta x + \gamma x)(\delta x + \iota x)(\varphi x + \iota x)(\rho x + \gamma x)(\sigma x + \gamma x) = f\]

... Implied: \(x^3 = 0\), Deduce and simplify...

Go back to Decision Level 3, simplify added clauses

Non-Chronological Backtrack
Backtrack to the MAX-level...

Conflict-inducement clause: 

Conflict assignments:

What assignments caused the conflict?
Deuce(); and continue....

Erase(); Backtrack to the "lowest possible level, Simplify();

If conflict(), Diagnose() from implication Graph

If unresolved clauses, continue from beginning

If solution found, Good, Go home!

Deduce() implications & construct Implication Graph

Simplify() the clauses &

Select an assignment

Generic Search for SAT