EX: Find $\text{Re}\left[\frac{6 + j3}{2 - j4} e^{jx}\right]$, (i.e., find the real part) where "x" is real.

ANS: $1.5 \cos(x + \pi/2)$

SOL’N: We may take one of several different approaches to convert the quantity inside the brackets into the form $a + jb$ (where $a$ is our final answer). We’ll take the approach of rationalizing the fraction.

\[
\text{Re}\left[\frac{6 + j3}{2 - j4} e^{jx}\right] = \text{Re}\left[\frac{6 + j3 \cdot 2 + j4}{2 - j4 \cdot 2 + j4} e^{jx}\right] \\
= \text{Re}\left[\frac{12 - 12 + j(24 + 6)}{2^2 + 4^2} e^{jx}\right] \\
= \text{Re}\left[\frac{j30}{20} e^{jx}\right]
\]

We now use Euler's formula to expand the complex exponential:

\[
= \text{Re}\left[\frac{j30}{20} \{\cos(x) + j\sin(x)\}\right] \\
= \text{Re}\left[-1.5\sin(x) + j1.5\cos(x)\right]
\]

Our final answer is the real part, which we may express in several ways.

\[
\text{Re}\left[\frac{6 + j3}{2 - j4} e^{jx}\right] = -1.5\sin(x) \text{ or }
\]

\[
\text{Re}\left[\frac{6 + j3}{2 - j4} e^{jx}\right] = 1.5\cos(x + \pi/2) = 1.5\cos(x + 90°)
\]

NOTE: A curious feature of this problem is that the fraction consisting of complex numbers is purely imaginary. We now examine this symbolically.

\[
k \cdot \frac{a + jb}{b - ja} = k \cdot \frac{j(b - ja)}{b - ja} = jk
\]

Whenever the numerator and denominator of a fraction have the above pattern, we will find that the result is purely imaginary. Note the necessary minus sign.