Now that we have voltages and currents which can be functions of time, it’s time to introduce the capacitor.

Fluid Model:

\[
\begin{align*}
\text{Capacitor} & \quad \text{+} \quad \text{Electrical equivalent:} \quad v_C \quad \text{like pressure} \quad \Delta P \\
\text{flow is like current} & \quad \text{+} \quad \text{como} \quad i_C
\end{align*}
\]

Capacitor: \( C = \varepsilon \frac{A}{d} = \frac{Q}{V} = \frac{dq}{dv} \)

Units: \( \text{farad} = \frac{\text{coul}}{\text{volt}} = \frac{\text{amp} \cdot \text{sec}}{\text{volt}} \)

Basic equations you should know:

\[
\begin{align*}
C &= \frac{Q}{V} \\
i_C &= C \frac{dv}{dt} \quad v_C
\end{align*}
\]

Energy stored in electric field: \( W_C = \frac{1}{2} C V_C^2 \)

Capacitor voltage cannot change instantaneously

Parallel: \( C_{\text{eq}} = C_1 + C_2 + C_3 + \ldots \)

Series: \( \frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \)

Capacitors are the only "backwards" components.

Sinusoids

\[
\begin{align*}
i_C(t) &= I_p \cos(\omega t) \\
v_C(t) &= \frac{1}{C} \int i_C \, dt = \frac{1}{C} \frac{1}{\omega} I_p \sin(\omega t) = \frac{1}{\omega} I_p \cos(\omega t - 90 \text{ deg})
\end{align*}
\]

Voltage "lags" current, makes sense, current has to flow in first to charge capacitor.

Steady-state or Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. That means the currents and voltages are no longer changing.

\[
\begin{align*}
\frac{dv_C}{dt} &= 0 \\
i_C &= C \frac{dv_C}{dt} = 0
\end{align*}
\]

no current means it looks like an open circuit.
Example

The voltage across a 0.5 µF capacitor is shown below. Make an accurate drawing of the capacitor current. Label the y-axis of your graph (I've already done the time-axis).

The accuracy of your plot at 0, 2, 6, and 8 ms is important, so calculate those values and plot or label them carefully. Between those points your plot must simply be the correct shape.

\[ C = 0.5 \text{ µF} \]

The curve is 2nd order

1 - 2ms: \[ i_C = C \frac{\Delta V}{\Delta t} = 0.5 \text{ µF} \cdot \frac{-4 \text{ V}}{2 \text{ ms}} = -1 \text{ mA} \]

2ms - 6ms: Initial slope is zero and the final slope is positive, so the current must be a triangle that starts at zero and ends at some height.

\[ \Delta v_C(t) = \frac{1}{C} \int_0^t i_C(t) \, dt \]

\[ 8 \text{ V} = \frac{1}{C} \left( \frac{4 \text{ ms} \cdot \text{height}}{2} \right) \]

height = \[ 8 \text{ V} \cdot \frac{C \cdot 2}{4 \text{ ms}} = 2 \text{ mA} \]

6ms - 8ms: Slope is zero, so the current must be zero.

RC first-order transient circuits

For all first order transients: \[ v_X(t) = v_X(\infty) + \left( v_X(0) - v_X(\infty) \right) e^{-\frac{t}{\tau}} \]

\[ i_X(t) = i_X(\infty) + \left( i_X(0) - i_X(\infty) \right) e^{-\frac{t}{\tau}} \]

Find the initial condition \[ v_X(0) \text{ or } i_X(0) \]

Find the capacitor voltage just before time \( t = 0 \), \( v_C(0-) \). This will be the same just after time \( t = 0 \), \( v_C(0+) \). The capacitor voltage cannot change instantly. (If the initial condition is zero then the capacitor will look like a short just after \( t = 0 \).) Use normal circuit analysis to find your desired variable: \[ v_X(0) \text{ or } i_X(0) \]

Find the final condition (“steady-state” or “forced” solution)

Capacitors are opens. Solve by DC analysis to find \[ v_X(\infty) \text{ or } i_X(\infty) \]

RC Time constant = \( \tau = R \text{Th} \cdot C \)

Curves
Electrical equivalent:

\[ i_L(t) = \frac{1}{L} \int_{t_0}^{t} v_L \, dt \]

Initial current

\[ i_L(t) = \frac{1}{L} \int_{0}^{t} v_L \, dt + i_L(0) \]

Or...

\[ \Delta i_L = \frac{1}{L} \int_{t_1}^{t_2} v_L \, dt \]

Energy stored in electric field:

\[ W_L = \frac{1}{2} L i_L^2 \]

Inductor current cannot change instantaneously

Units:

\[ \text{henry} = \frac{\text{volt} \cdot \text{sec}}{\text{amp}} \]

\[ mH = 10^{-3} \cdot H \quad \mu H = 10^{-6} \cdot H \]

Series:

\[ L_{eq} = \frac{1}{L_1 + L_2 + L_3 + \ldots} \]

Parallel:

\[ L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \ldots} \]

Sinusoids

\[ i_L(t) = I_p \cos(\omega t) \]

\[ v_L(t) = L \frac{d}{dt} i_L = L \omega \left( -I_p \sin(\omega t) \right) = L \omega I_p \cos(\omega t + 90\deg) \]

Voltage “leads” current, makes sense, voltage has to present to make current change, so voltage comes first.