DEF: N-dimensional Fourier series for \( f(x) \) on domain \( \left[-\frac{1}{2}, \frac{1}{2}\right]^N \) (or other unit hypercube such as \( [0,1]^N \)) =
\[
 f(\vec{x}) = \sum_{\vec{n}} \left[ a_{\vec{n}} \sqrt{2} \cos(2\pi \vec{n} \cdot \vec{x}) + b_{\vec{n}} \sqrt{2} \sin(2\pi \vec{n} \cdot \vec{x}) \right]
\]
where \( \vec{n} \) ranges over all vectors of form \((n_1, \ldots, n_N)\) with integer entries and first nonzero entry restricted to positive values.

NOTE: The restriction that the first nonzero entry in \( \vec{n} \) be positive avoids having essentially the same cosine or sine term appear twice in the series owing to the similarity of sinusoids with positive and negative arguments:
\[
\cos(x) = \cos(-x) \quad \text{and} \quad \sin(x) = \sin(-x)
\]
In other words, the restriction on \( \vec{n} \) eliminates \( -\vec{n} \) from the series.

NOTE: \( \sqrt{2} \cos(2\pi \vec{n} \cdot \vec{x}) = \sqrt{2} \cos(2\pi[n_1x_1 + n_2x_2]) \) in two dimensions.

NOTE: \( \sqrt{2} \cos(2\pi n_1x_1 + n_2x_2) \) [or \( \sqrt{2} \sin(2\pi n_1x_1 + n_2x_2) \)] resembles water waves with crests lying on parallel lines.

TOOL: \( \vec{n} \) is \( \perp \) (is perpendicular to) the wave crests of \( \sqrt{2} \cos(2\pi \vec{n} \cdot \vec{x}) \)

TOOL: The spacing, \( d \), of wave crests of \( \sqrt{2} \cos(2\pi \vec{n} \cdot \vec{x}) \) is found by solving the following equation:
\[
\vec{n} \cdot \frac{\vec{n}}{|\vec{n}|} = 1 \quad \text{(so argument of cos() = 2π; crests occur where cos() = 1)}
\]
or
\[
d = \frac{1}{|\vec{n}|}
\]