EX: Using Matlab® or pencil and paper, make an accurate plot of a standard gaussian distribution and answer the following questions:

a) What is the value of \( f(x) \) at \( x = 0 \)?
b) At what value of \( x \) does \( f(x) = 0.5 \)?
c) Estimate by eye the value of \( x \) for which \( F(x) = 0.25 \).
d) Use a table of area under the normal (i.e., gaussian) curve to find the value of \( x \) for which \( F(x) = 0.25 \).
e) Use a table of area under the normal (i.e., gaussian) curve to find \( P(1 \leq x \leq 2) \).

SOL’N: a) The standard gaussian has \( \mu = 0 \) and \( \sigma^2 = 1 \):

\[
f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}
\]

The value of \( f(x) \) at \( x = 0 \) is the constant term since \( e^0 = 1 \):
\[
f(0) = \frac{1}{\sqrt{2\pi}} \approx 0.3989 = 0.4
\]

b) From the plot, we observe that \( f(x) \) never reaches a value of 0.5.

c) \( F(x) \) is the cumulative distribution function, which is equal to the area under the probability function to the left of \( x \). Thus, we are looking for the value of \( x \) where the area to the left of \( x \) is 1/4 of the total area of the probability density function, (since the total area under the probability density function is equal to one).

The author's estimate is \( x \approx -3/4 \).

d) Since we have a standard gaussian, we may use a table for the area under a standard gaussian directly. Note that such tables give values of \( F(x) \). We use the table in reverse, however. We look for the value of \( F(x) = 0.25 \) in the table and then look at the value of \( x \) that corresponds to that \( F(x) \). The value we obtain is \(-0.675\) to three significant figures.

e) \( P(1 \leq x \leq 2) = P(x \leq 2) - P(1 \leq x) = F(x = 2) - F(x = 1) \). Using a table for the area under a standard gaussian, we have

\[ F(x = 2) = 0.9772 \] and \( F(x = 2) = 0.8413 \).

Thus,

\[ P(1 \leq x \leq 2) = 0.9772 - 0.8413 = 0.1359. \]