ex:

\[
\begin{align*}
v_1 & = V_1 \\
v & = V \\
\frac{1}{2} \mu F + \frac{1}{8} \mu F & = V \\
2 \mu F & + v_2 = V \\
v_1(t=0) = -10 V & \\
v_2(t=0) = -5 V \\
i & = 240 e^{-10t} \mu A \\
\end{align*}
\]

Calculate total energy trapped in C's as \( t \to \infty \).

Hint: Don't combine C's in series - find energy for each C and sum them.

Note: The energy trapped in C's refers to energy that we cannot extract from the C's when the total \( V \) across the two C's in series is 0V.

For example:

\[
\begin{align*}
V & = 1V \\
+ & \\
\begin{array}{c}
\text{Here we have:} \\
v_{1v2} = v = 0V \\
\text{but each C has} \\
\text{\( V \to 0 \) across it.}
\end{array}
\end{align*}
\]

If we connect an \( R \) across the terminals, we will get no current flow because \( V = 0V \).

Thus, we cannot access the "trapped" energy.

The trapped energy is \( w_{t1} + w_{t2} = \frac{1}{2} \mu F (1V)^2 + \frac{1}{8} \mu F (2V)^2 \)

\[
= \frac{1}{2} \mu F + \frac{1}{4} \mu F = 5 \mu F.
\]

(Note that for series combination we would get \( \frac{1}{2} C_{eq} V^2 = \frac{1}{2} C_{eq} \cdot 0^2 = 0 J \).

Note: The energy stored in C's refers to energy that we can extract from the C's by connecting a circuit to the two terminals having series C's between.

For the stored energy, we get the correct answer if we use \( C_{eq} = \frac{1}{2} C_1 + \frac{1}{2} C_2 = C_1 C_2 / (C_1 + C_2) \) (see next page). 

Moral: We can use \( C_{eq} \) if all we care about are the \( i \) and \( v \) at terminals, (or the energy we can extract from the terminals).

We now consider this energy issue in detail before finally solving the problem.
Assume no energy is trapped when $v=0$.

Now let current $i$ flow until time $t$.

$$i = C \frac{dv}{dt} \Rightarrow \int i dt = C \cdot v_1$$

for each $C_i = C_1$ or $C_2$.

Both $C_1$s see same $i$ (since in series), so

$$\int i dt$$

same for both $C_1$ and $C_2$.

$$\therefore C_1 v_1 = C_2 v_2 \quad \text{or} \quad v_2 = \frac{C_1}{C_2} v_1$$

Now, $C_{eq} = C_1 \parallel C_2 = \frac{C_1 C_2}{C_1 + C_2}$.

$$\frac{1}{2} C_{eq} v^2 = \frac{1}{2} C_{eq} (v_1 + v_2)^2 = \frac{1}{2} \left( \frac{C_1 C_2}{C_1 + C_2} \right) \left( v_1 + \frac{C_1}{C_2} v_1 \right)^2$$

$$= \frac{1}{2} \left( \frac{C_1 C_2}{C_1 + C_2} \right) \left( \frac{(C_1 + C_2) v_1}{C_2} \right)^2$$

$$= \frac{1}{2} \left( \frac{C_1}{C_2} \right) (C_1 + C_2) v_1^2$$

$$= \frac{1}{2} C_{eq} v^2 \checkmark$$

If there is trapped energy, then when $v=0$ we have $v_1 = v_0$ and $v_2 = -v_0$ for some voltage $v_0$.

If we let current $i$ flow and store energy then we have

$$\int i dt = C_1 (v_1 - v_0) = C_2 (v_2 + v_0), \quad \text{or} \quad v_2 = \frac{1}{C_2} \left[ (C_1 + C_2) v_0 \right]$$

The stored energy on $C_{eq}$ is

$$\frac{1}{2} C_{eq} v^2 = \frac{1}{2} C_{eq} (v_i + v_0)^2$$

This is equal to the total energy for $C_1$ and $C_2$ minus the trapped energy. In other words the stored energy is

$$\frac{1}{2} C_1 v_1^2 + \frac{1}{2} C_2 v_2^2 = \left( \frac{1}{2} C_1 v_1^2 + \frac{1}{2} C_2 v_0^2 \right) = \left( \frac{1}{2} C_{eq} v_i^2 \right)$$

total energy trapped energy stored energy
We now find \( v_1(t \to \infty) \) and \( v_2(t \to \infty) \).

\[
i = 240 \, e^{-10t} \, \mu A = C_1 \, \frac{dv_1}{dt} = C_2 \, \frac{dv_2}{dt} \quad \text{same i flows thru}
\]

\[
\int_{t=0}^{t=\infty} i(t) \, dt = \int_{t=0}^{t=\infty} \frac{240 \, e^{-10t}}{C_1} \, dv_1 = \int_{t=0}^{t=\infty} C_2 \, dv_2
\]

\[
\left. \frac{240 \, e^{-10t}}{C_1} \right|_{t=0}^{t=\infty} = C_1 \left[ v_1(t=\infty) - v_1(t=0) \right] = C_2 \left[ v_2(t=\infty) - v_2(t=0) \right]
\]

\[
-24 \, e^{-10t} \, \mu A \, s = 2 \, \mu F \left[ v_1(t=\infty) - 10V \right] = 8 \, \mu F \left[ v_2(t=\infty) - 5V \right]
\]

\[
24 \, \mu A \, s = 2 \, \mu F \left[ v_1(t=\infty) + 10V \right] = 8 \, \mu F \left[ v_2(t=\infty) + 5V \right]
\]

\[
v_1(t=\infty) = \frac{24 \, \mu A \, s}{2 \, \mu F} = 12V - 10V = 2V
\]

\[
v_2(t=\infty) = \frac{24 \, \mu A \, s}{8 \, \mu F} = 3V - 5V = -2V
\]

Energy trapped \( E_{\text{trapped}} = \frac{1}{2} C_1 v_1^2(t=\infty) + \frac{1}{2} C_2 v_2^2(t=\infty) \)

\[
= \frac{1}{2} 2 \, \mu F (2V)^2 + \frac{1}{2} 8 \, \mu F (-2V)^2
\]

\[
= 4 \, \mu J + 16 \, \mu J
\]

\[
= 20 \, \mu J
\]

Note: The given \( i = 240 \, e^{-10t} \) leaves us with
\[
v = v_1(t=\infty) + v_2(t=\infty) = 0V. \quad \text{We must have}
\]
\[
v = 0V \quad \text{for the calculation of the trapped energy.}
\]

Otherwise, we also have stored energy \( = \frac{1}{2} C_2 \, v^2 \).

A different \( i \) might not have resulted in \( v = 0 \)
as \( t \to \infty \).