ex:

No energy stored in $L_1$ and $L_2$ when switch opens.

a) Find $i_1(t \geq 0)$ and $i_2(t \geq 0)$.

b) Find $i_1(t \to \infty)$ and $i_2(t \to \infty)$.

\[ a) \quad i_1(t \geq 0) = I_g \frac{L_2}{L_1 + L_2} \left( 1 - e^{-t/\tau} \right) \quad \text{where} \quad \tau = L_1 || L_2 \]

\[ i_2(t \geq 0) = I_g \frac{L_1}{L_1 + L_2} \left( 1 - e^{-t/\tau} \right) \]

\[ b) \quad i_1(t \to \infty) = I_g \frac{L_2}{L_1 + L_2} \quad i_2(t \to \infty) = I_g \frac{L_1}{L_1 + L_2} \]

soln: a) Take Thévenin equivalent of $I_g$ and $R_g$ on left.

Solve for $v$ across $L'$s by replacing $L'$s with equivalent $L$.

\[ \text{Circuit for } t \geq 0 \]

$L'$s in parallel give $L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$.

$L'$s in parallel are like $R'$s in parallel in terms of the formula we use.

Now we use the general solution for $v(t \geq 0)$:

\[ v(t \geq 0) = v(t \to \infty) + \left[ v(0^+) - v(t \to \infty) \right] e^{-t/\left( L_{eq} R_g \right)} \]

To find $v(0^+)$, we use $i_1(0^+) = i_1(0^-)$ and $i_2(0^+) = i_2(0^-)$. But $i_1(0^-) = i_2(0^-) = 0$ since no energy is stored in $L_1$ and $L_2$ at $t = 0$. 
a) cont.

Since \( i_1(0^+) \) and \( i_2(0^+) = 0 \), we must have no current through \( R_g \) at \( t = 0^+ \).

\[ \therefore \text{At } t=0^+, \text{ we have no } v \text{ drop across } R_g. \]

\[ \therefore v(t=0^+) = V_{Th} = I_g \cdot R_g \]

For \( v(t \to \infty) \) we observe that the \( L \)'s act like wires, and \( v(t \to \infty) = 0 \).

Plugging into the general soln gives

\[ v(t \geq 0) = I_g \cdot R_g \cdot e^{-t/(\text{Log } R_g)} \]

Note: The time constant for circuit with \( L \) and \( R \) is \( \text{Log } R \). Thevenin equivalent always gives the needed \( R \).

Now we can also write down a formula for \( i(t) = i_1(t) + i_2(t) \) for \( t \geq 0 \):

\[ i(t \geq 0) = i(t \to \infty) + [i(0^+) - i(t \to \infty)] e^{-t/(\text{Log } R_g)} \]

Note: All \( i \)'s and \( v \)'s have same time constant.

We know \( i(0^+) = i_1(0^+) + i_2(0^+) = i_1(0^-) + i_2(0^-) = 0 \).

At \( t \to \infty \), the \( L \)'s act like wires, giving \( i = I_g \).

\[ \therefore i(t \geq 0) = I_g \left[ 1 - e^{-t/(\text{Log } R_g)} \right] \]

Now we determine how \( i(t \geq 0) \) is divided between the two \( L \)'s to give \( i_1(t \geq 0) \) and \( i_2(t \geq 0) \).
a) cont.

Since both L's have same V across them, we have

\[ v = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} \]

\[ \therefore \frac{di_1}{dt} = \frac{L_2}{L_1} \frac{di_2}{dt} \]

Now we calculate currents:

\[ i_1(t) = \int \frac{di_1}{dt} dt = \int \frac{L_2}{L_1} \frac{di_2}{dt} dt = \frac{L_2}{L_1} \int di_2 \]

or \[ i_1(t) = \frac{L_2}{L_1} i_2(t) \]

Also, \[ i_1(t) + i_2(t) = i(t) \].

Solving these two eqns gives

\[ i_1(t) = \frac{L_2}{L_1 + L_2} i(t) \]

\[ i_2(t) = \frac{L_1}{L_1 + L_2} i(t) \]

Thus,

\[ i_1(t\geq0) = I_g \frac{L_2}{L_1 + L_2} \left( 1 - e^{-t/(\text{Log} / R_g)} \right) \]

\[ i_2(t\geq0) = I_g \frac{L_1}{L_1 + L_2} \left( 1 - e^{-t/(\text{Log} / R_g)} \right) \]

b) At \( t \to \infty \) we have \( e^{-t/(\text{Log} / R_g)} \to 0 \).

\[ \therefore i_1(t\to\infty) = I_g \frac{L_2}{L_1 + L_2} \]

\[ i_2(t\to\infty) = I_g \frac{L_1}{L_1 + L_2} \]