EX:

\[ v_s = 15 \text{ V} \]

\[ R_1 = 130 \Omega \]

\[ R_2 = 160 \Omega \]

\[ C = 500 \text{ pF} \]

\[ L_1 = 5 \mu \text{H} \]

After being open for a long time, the switch closes at \( t = 0 \).

Find \( v(t) \) for \( t > 0 \).

Sol'n: First, we find the characteristic roots for the circuit. After \( t = 0 \) we have a series RLC circuit with a voltage source. \( R_1 \) is bypassed so we use \( R = R_2 \).

For series RLC, \( \alpha = \frac{R}{2L} \), \( \omega_0^2 = \frac{1}{LC} \)

\[ \alpha = \frac{160 \Omega}{2 \cdot 5 \mu \text{H}} = \frac{1}{2 \cdot 500 \text{ pF}} \]

\[ \alpha = 16 \text{ M} \text{r/s}, \quad \omega_0^2 = 400 \text{ M}^2(\text{r/s})^2 \]

\[ \omega_0 = 20 \text{ M} \text{r/s} \]

\( \omega_0 > \alpha \) so we have underdamped case:

\[ \omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{(20 \text{ M})^2 - (16 \text{ M})^2} \text{ r/s} \]
\[ \omega_d = 12 \, \text{M r/s} \]

Note: \( \omega_d > \alpha \) can happen but \( \omega_d < \omega_0 \) always.

Our solution form is

\[ v(t) = A_1 e^{-\alpha t} \cos \omega_d t + A_2 e^{-\alpha t} \sin \omega_d t + A_3. \]

Second, we find \( A_3 = v(t \to \infty) \).

As \( t \to \infty \), \( C = \text{open} \quad L = \text{wire switch closed} \)

\[ v(t \to \infty) = -15V \]

\[ \therefore A_3 = -15V \]

Third, we find \( i_L(t=0^-) \) and \( v_c(t=0^-) \) as a precursor to finding \( v(0^+) \) and \( \left. \frac{dv}{dt} \right|_{t=0^+} \).

At \( t=0^- \), \( C = \text{open} \quad L = \text{wire switch open} \)

\[ \text{model: } V_{15V} \quad (t=0^-) \]

\[ \text{Source not connected} \quad \text{so} \quad i_L(0^-) = 0 \quad \text{and} \quad v_c(0^-) = 0. \]
Fourth, we have $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$ since these are energy variables ($w = \frac{1}{2}Li^2$ and $w = \frac{1}{2}CV^2$) that cannot change instantly.

At $t=0^+$, we model $L$ as a current source with value $i_L(0^+)$ and $C$ as a voltage source with value $v_C(0^+)$. 

Fifth, we solve the circuit to find $v(0^+)$. Here, we have that $v(0^+) = 0V$ without doing any additional work.

For the form of sol'n we are using, we have

$$v(0^+) = \left. A_1e^{-\alpha t} \cos (\omega dt) + A_2e^{-\alpha t} \sin (\omega dt) + A_3 \right|_{t=0^+}$$

$$\equiv A_1 e^{-\alpha 0^+} \cos (0) + A_2 e^{-\alpha 0^+} \sin (0) + A_3$$

$$\equiv A_1 \cdot 1 \cdot 1 + A_2 \cdot 1 \cdot 0 + A_3$$

$$\equiv A_1 + A_3$$

Equate the known value of $v(0^+) = 0V$ with the symbolic sol'n, we conclude that:
0V = A_1 + A_3 = A_1 - 15V or A_1 = 15V

Sixth, we use the circuit model at t=0^+ to find \( \frac{d}{dt} v(t) \bigg|_{t=0^+} \).

The method we use to find any derivative value at t=0^+ is to write an expression for \( v(t) \) in terms of only \( i_L(t) \) and \( v_c(t) \) plus component values.

Here, we have the simple result that

\[ v(t) = v_c(t) \]

Now we differentiate this entire eqn:

\[ \frac{d}{dt} v(t) = \frac{d}{dt} v_c(t) \]

From \( i_c(t) = C \frac{dv_c(t)}{dt} \) we have \( \frac{d}{dt} v_c(t) = \frac{i_c(t)}{C} \).

(Although we don't require it here, we also have \( v_L(t) = L \frac{di_L(t)}{dt} \) or \( \frac{d}{dt} i_L(t) = \frac{v_L(t)}{L} \).)

Thus \( \frac{d}{dt} v(t) \bigg|_{t=0^+} = \frac{d}{dt} v_c(t) \bigg|_{t=0^+} = \frac{i_c(t)}{C} \bigg|_{t=0^+} \).

We use our model for t=0^+ to find \( i_c(t=0^+) \). (See above.) From the model, \( i_c(0^+) = 0 \)A since \( C \) is in series with a \( 0 \)A source.
\[ \frac{dv(t)}{dt} \bigg|_{t=0^+} = \frac{i_c(t)}{c} \bigg|_{t=0^+} = 0A = 0 \text{ V/s} \]

Equating this known value of \( \frac{dv(t)}{dt} \) with \( \frac{d}{dt} \) of the symbolic solution, \( \frac{d}{dt} \bigg|_{t=0^+} \)

we have

\[ OV = \frac{d}{dt} v(t) \bigg|_{t=0^+} = \frac{d}{dt} \left[ A_1 e^{-\kappa t} \cos(\omega_d t) + A_2 e^{-\kappa t} \sin(\omega_d t) \right] \bigg|_{t=0^+} \]

\[ n = A_1 (-\kappa) e^{-\kappa t} \cos(\omega_d t) + A_1 e^{\omega_d [-\sin(\omega_d t)]} \bigg|_{t=0^+} \]

\[ + A_2 (-\kappa) e^{-\kappa t} \sin(\omega_d t) + A_2 e^{\omega_d \cos(\omega_d t)} \bigg|_{t=0^+} \]

\[ n = A_1 (-\kappa) \cdot 1 + A_1 \cdot 1 \cdot \omega_d \cdot 0 \]

\[ A_2 (-\kappa) \cdot 1 + A_2 \cdot 1 \cdot \omega_d \cdot 1 \]

\[ OV = A_1 (-\kappa) + A_2 \omega_d \]

Thus, \( A_2 = \frac{A_1 \kappa}{\omega_d} = \frac{15V \cdot 16 \text{ Ms}}{12 \text{ Ms}} = 20V \)

\[ v(t > 0) = 15Ve^{-16Mt} \cos(12Mt) \]

\[ + 20Ve^{-16Mt} \sin(12Mt) \]

\[ -15V \]

Check: \( v(0^+) = 15V - 15V = 0V \) \checkmark

When \( \frac{dv}{dt} \bigg|_{t=0^+} = 0 \) we should have \( A_1 \kappa = A_2 \omega_d \)