In the above circuit, the switch moves from a to b at t = 0.
Given that no energy is stored in the inductor at time t = 0, find a numerical expression for v(t) for t > 0.

Sol'n: First, we find the initial conditions, \( i_L(0^-) \) and \( v_C(0^-) \).

Since there is no energy stored in the inductor and energy \( w = \frac{1}{2} L i_L^2(0^-) \), we have \( i_L(0^-) = 0A \).

The capacitor will charge to 40V measured with + on the left side of C: \( v_C(0^-) = 40V \).

Since the inductor current and capacitor voltage do not change instantly, we have
\[
  i_L(0^+) = i_L(0^-) = 0A, \quad v_C(0^+) = v_C(0^-) = 40V
\]

Second, we calculate the characteristic roots of the circuit for \( t > 0 \).
For $t > 0$, we have a series RLC circuit consisting of $R = 130 \text{k} \Omega$, $L = 250 \text{mH}$, and $C = \frac{1}{1024} \text{\mu F}$.

The characteristic roots are

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

where

$$\alpha = \frac{R}{2L} \quad \text{and} \quad \omega_0^2 = \frac{1}{LC}$$

Plugging in values, we have

$$\alpha = \frac{130 \text{k} \Omega}{2 \cdot 250 \text{mH}} = 260 \text{ k rad/s}$$

$$\omega_0^2 = \frac{1}{250 \text{mH} \cdot \frac{1}{1024} \text{\mu F}} = 4096 \text{ M rad}^2/\text{s}^2$$

or $(64 \text{k rad/s})^2$

$$s_{1,2} = -260k \pm \sqrt{260^2 k^2 - 64^2 k^2} \quad \text{rad/s}$$

$$s_{1,2} = -260k \pm 252k \quad \text{rad/s}$$

$$s_{1,2} = -8k \text{ rad/s} \quad \text{and} \quad -512k \text{ rad/s}$$

Third, we use the appropriate form of general solution for the over-damped case:

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + A_3$$
Fourth, we find $A_3$ as the final value for the circuit. As $t \to \infty$, we assume currents and voltages reach constant values. In other words, derivatives equal zero. Thus,

$$v_L = L \frac{di_L}{dt} = 0 \text{ V} \quad \text{and} \quad i_c = C \frac{dv_c}{dt} = 0 \text{ A}$$

Since $v_L = 0 \text{ V}$, $L$ acts like a wire. Since $i_c = 0 \text{ A}$, $C$ acts like an open circuit.

For $t \to \infty$, our circuit model is

```
   C
```

```
+V(t->\infty)---
|                  |
|                  |
|                  |
+------------------+
   L   R=130 k \Omega
```

Since $C$ is an open circuit and there is no power source, $V(t \to \infty) = 0 \text{ V}$.

$$\therefore A_3 = 0 \text{ V}$$

Fifth, we find $V(t=0^+)$ by modeling $L$ as a current source with value $i_L(t=0^+)$ and $C$ as a voltage source with value $V_c(t=0^+)$. $V_c(0^+) = 40 \text{ V}$

```
  +-------------------+
  |                   |
  +-------------------+
  |                   |
     +v(0^+)-
```

```
+-------------------+
  |                   |
  +-------------------+
  |                   |
     i_L(0^+)=0 \text{ A}
```

$= \text{open circuit}$
Since \( i_L(0^+) = 0 \) A, there is no current flowing in \( R \). Thus, \( v(0^+) = i \cdot R = 0 \) V.

\[ v(0^+) = 0 \text{ V} \]

Sixth, we find \( \frac{dv(t)}{dt} \bigg|_{t=0^+} \) by writing \( v(t) \) in terms of \( i_L(t) \) and \( v_c(t) \). Here,

\[ v(t) = i_L(t) \cdot R. \]

\[ \frac{dv(t)}{dt} = \frac{d[i_L(t) \cdot R]}{dt} = \frac{di_L(t)}{dt} \cdot R \]

We now write \( \frac{di_L(t)}{dt} \) in terms \( v_L(t) \):

\[ v_L(t) = L \frac{di_L(t)}{dt} \quad \text{or} \quad \frac{di_L(t)}{dt} = \frac{v_L(t)}{L} \]

Thus, we have

\[ \frac{dv(t)}{dt} = \frac{v_L(t)}{L} \cdot R. \]

Evaluating at \( t=0^+ \):

\[ v_c(0^+) = 40 \text{ V} \]

We have \( v_L(0^+) = v_c(0^+) = 40 \text{ V} \).
We have

\[ \frac{dv(t)}{dt} \bigg|_{t=0^+} = v_L(0^+) R = \frac{40v \cdot 130k\Omega}{250m\Omega} \]

\[ \frac{dv(t)}{dt} \bigg|_{t=0^+} = 20.8 \text{ MV/s} \]

Seventh, we match symbolic initial values with numerical values.

\[ v(0^+) = A_1 + A_2 = 0v \]

\[ \frac{dv(t)}{dt} \bigg|_{t=0^+} = A_1 s_1 + A_2 s_2 = 20.8 \text{ MV/s} \]

From the first equation, \( A_2 = -A_1 \).

\[ A_1 s_1 - A_1 s_2 = A_1 (s_1 - s_2) = 20.8 \text{ MV/s} \]

\[ A_1 = \frac{20.8 \text{ MV/s}}{s_1 - s_2} = \frac{20.8 \text{ MV/s}}{-8(-512) \text{ k r/s}} \]

\[ A_1 = 41.3 V \quad \Rightarrow \quad A_2 = -41.3 V \]

Thus, we have our final answer:

\[ v(t) = 41.3 e^{-8kt} - 41.3 e^{-512kt} V \]