Ex:

\[ i_s = 4 \text{ mA} \]

\[ \begin{array}{c}
R_1 = 150 \Omega \\
C = 2 \mu F \\
R_2 = 100 \Omega \\
L \\
i_L(t)
\end{array} \]

After being closed for a long time, the switch opens at \( t = 0 \).

a) If \( L = 125 \text{ mH} \), find the characteristic roots, \( s_1 \) and \( s_2 \), for the above circuit.

b) If \( L = 11.834 \text{ mH} \), find the damping frequency, \( \omega_d \).

c) Find the value of \( L \) that makes the circuit critically damped.

SOL'N: a) After \( t=0 \) we have parallel RLC.

From the differential eq'n for the parallel RLC with \( Ae^{st} \) substituted for the sol'n, we get the characteristic eq'n for the circuit:

\[ s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0 \]

We get the characteristic roots by solving this quadratic eq'n:

\[ s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \]

where \( \alpha \equiv \frac{1}{2RC} \quad \omega_0^2 \equiv \frac{1}{LC} \)

Now we plug in values for components.

\( R = R_2 = 100 \Omega \quad L = 125 \text{ mH} \) (for part a) \( C = 2 \mu F \)
\[ \alpha = \frac{1}{2 \cdot 100 \Omega \cdot 2 \mu F} = \frac{1}{400} \text{ M rad/s} = \frac{1}{1k} \text{ M rad/s} \]

\[ \alpha = 2.5 \ \text{k rad/s} \]

\[ \omega_0^2 = \frac{1}{125 \text{ mH} \cdot 2 \mu F} = \frac{1}{250 \text{ ns}} \cdot \frac{(\text{rad/s})^2}{1k} \]

\[ \omega_0^2 = 4 \text{ M (rad/s)}^2 \quad \text{or} \quad \omega_0 = 2k \text{ rad/s} \]

\[ \therefore s_{1,2} = -2.5k \pm \sqrt{(2.5k)^2 - (2k)^2} \text{ rad/s} \]

\[ = -2.5k \pm 1.5k \ \text{ rad/s} \]

\[ s_{1,2} = -4k \text{ and } -1k \ \text{ rad/s} \]

Note: Real part of s is always \( \geq 0 \) for an RLC circuit.

b) For \( L = 11.834 \text{ mH} \) we have different \( \omega_0^2 \):

\[ \omega_0^2 = \frac{1}{LC} = \frac{1}{11.834 \text{ mH} \cdot 2 \mu F} = \frac{42.25}{\text{m rad}} \]

\[ \omega_0^2 = 42.25 \text{ M rad/s} \quad \text{or} \quad \omega_0 = 6.5k \text{ rad/s} \]

Note: Changing \( L \) does not change \( \alpha \) for a parallel RLC, but it would change \( \alpha \) for a series RLC where \( \alpha = R/2L \).

Here, \( s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \) with \( \omega_0 > \alpha \).
Since \( \omega_0 > \kappa \), we get complex roots:
\[
S_{1,2} = -2.5k \pm \sqrt{(2.5k)^2 - (6.5k)^2} \text{ rad/s}
\]
\[
S_{1,2} = -2.5k \pm j6.0k \text{ rad/s}
\]

We define the damping frequency to be the magnitude of the \( \sqrt{ } \) term, or
\[
\omega_d = \sqrt{\omega_0^2 - \kappa^2} \quad \text{(order of \( \omega_0, \kappa \) is reversed)}
\]
\[
\omega_d = \sqrt{(6.5k)^2 - (2.5k)^2} \text{ rad/s}
\]
\[
\omega_d = 6k \text{ rad/s}
\]

c) The circuit is critically damped when \( S_1 = S_2 \), i.e. when \( \sqrt{ } = 0 \) or \( \kappa = \omega_0 \).

Since \( \kappa = 2.5k \text{ rad/s} \) doesn't change with \( L \), we must have \( \omega_0 = 2.5k \text{ rad/s} \).

\[
1 = \frac{2.5k}{\sqrt{LC}} \Rightarrow \frac{1}{LC} = (2.5k/r)^2
\]

or
\[
L = \frac{1}{(2.5k)^2} = \frac{1}{6.25k^2} = \frac{1}{6.25 \cdot 2}
\]
\[
L = 80 \text{ mH}
\]