Ex:

![RLC Circuit Diagram]

After being open for a long time, the switch closes at $t = 0$.

The inductor carries no current at time $t = 0^-$.

(a) Give expressions for the following in terms of $i_g$, $R$, $L$, and $C$:

$$i(t = 0^+) \quad \text{and} \quad \frac{di(t)}{dt} \bigg|_{t=0^+}$$

(b) Find the numerical values of $L$ and $R$ given the following information:

$$C = 5 \ \mu F \quad s_1 = -10k \ \text{rad/s} \quad s_2 = -40k \ \text{rad/s}$$

**Solution:**

We find initial conditions by starting at $t = 0^-$. $(L$ acts like wire, $C$ acts like open$)$

$$i_g \uparrow \quad \text{and} \quad V_C(0^-) = i_g R$$

$$i_L(0^-) = 0 \ A \quad \text{since prob says so}$$

We only find the values of these energy variables at $t=0^-$ because they will not change instantly when we close the switch.
At $t=0^+$: (model $L$ as $i_{src}$, $C$ as $v_{src}$)

$$\begin{align*}
    \frac{di}{dt} + \frac{1}{C} \frac{dv}{dt} &= 0 \\
    \frac{dv}{dt} &= \frac{1}{R} \frac{di}{dt} \\
    v_{c}(0^+) &= v_{c}(0^-) = i_{g}R \\
    i_{L}(0^+) &= i_{L}(0^-) = 0A
\end{align*}$$

Since $i_{L}(0^+) = 0A$, the $L$ acts like it is not there still.

The current thru $R$ is $v_{c}(0^+)/R = i_{g}$.

Thus, no current is left to flow thru $C$, and $i(0^+) = 0A$.

To find $\frac{di}{dt}$ at $t=0^+$, we start by writing $i$ in terms of $i_{L}$ and $v_{c}$.

(Don't plug in $t=0^+$ or take $d/dt$ yet.)

Summing current out of the top wire:

$-i_{g} + i_{R} + i + i_{L} = 0A$

Replacing $i_{R}$ with $v_{c}/R$, we have

$i = i_{g} - \left( \frac{v_{c}}{R} + i_{L} \right)$.

Taking $\frac{d}{dt}$ of both sides:

$$\frac{d}{dt} + \frac{1}{C} \frac{d}{dt} = -\frac{1}{R} \frac{dv}{dt} - \frac{di}{dt}$$

We use $\frac{dv}{dt} = \frac{i_{c}}{C}$ and $\frac{di}{dt} = \frac{v_{L}}{L}$.
\[
\frac{di}{dt} = -\frac{1}{RC} i_c - \frac{v_L}{L}
\]

\[
\therefore \left. \frac{di}{dt} \right|_{t=0^+} = -\frac{1}{RC} i_c(0^+) - \frac{v_L(0^+)}{L}
\]

Returning to our circuit for \( t = 0^+ \), we have \( i_c(0^+) = i(0^+) = 0 A \).

Also, \( v_L(0^+) = v_c(0^+) = i g R \).

\[
\left. \frac{di}{dt} \right|_{t=0^+} = -\frac{ig R}{L}
\]

b) We always have \( s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \).

We have a parallel RLC with \( \alpha = \frac{1}{2RC} \).

For any simple RLC, \( \omega_0^2 = \frac{1}{LC} \).

From the above, we use

\[
s_1 + s_2 = -2\alpha = -\frac{1}{2RC}
\]

or \( -10k \Omega - 40k \Omega = -\frac{1}{5 \mu F} \)

or \( R = \frac{-1}{-50k \cdot 5 \mu} = \frac{1}{2.5 \Omega} = 0.4 \Omega \)

\[
R = 0.4 \Omega \quad \text{(and } \alpha = 25k \Omega) \]
To find $L$, we use

$$s_1 \cdot s_2 = \left( -\kappa + \sqrt{\kappa^2 - \omega_0^2} \right) \left( -\kappa - \sqrt{\kappa^2 - \omega_0^2} \right)$$

$$= (-\kappa)^2 - \sqrt{\kappa^2 - \omega_0^2}^2$$

$$= \kappa^2 - (\kappa^2 - \omega_0^2)$$

$$= \omega_0^2$$

$$= \frac{1}{LC}$$

$$\therefore L = \frac{1}{s_1 \cdot s_2 C}$$

$$L = \frac{1}{(-10k)(-40k)\ 5\mu}$$

$$L = \frac{1}{2000}$$

$$L = 500\ \mu\text{H}$$