**Ex:** The following data have been obtained for the lifetimes of three different printer models, (values are in weeks):

\[
\begin{align*}
    x_{11} &= 98 & x_{12} &= 81 & x_{13} &= 90 & x_{14} &= 88 & x_{15} &= 83 \\
    x_{16} &= 86 & x_{17} &= 99 & x_{18} &= 86 & x_{19} &= 99 \\
    x_{21} &= 66 & x_{22} &= 67 & x_{23} &= 67 & x_{24} &= 59 & x_{25} &= 65 \\
    x_{26} &= 76 & x_{27} &= 68 & x_{28} &= 70 & x_{29} &= 74 \\
    x_{31} &= 107 & x_{32} &= 107 & x_{33} &= 124 & x_{34} &= 110 & x_{35} &= 112 \\
    x_{36} &= 115 & x_{37} &= 124 & x_{38} &= 123 & x_{39} &= 113
\end{align*}
\]

Each observation, \(x_{ij}\), was obtained by averaging 30 values. Thus, the data is assumed by the central limit theorem to be close to gaussian (normal).

Use Bartlett's test of variances, [1], to determine whether to accept the following null hypothesis, \(H_0\), or the alternate hypothesis, \(H_1\), at the 1% significance level.

\[
\begin{align*}
    H_0: & \quad \sigma_1^2 = \sigma_2^2 = \sigma_3^2 \\
    H_1: & \quad \text{At least one of the means is not equal to the others}
\end{align*}
\]

**Sol'n:** We first find the calculated sample means:

\[
\bar{x}_{i.} = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij}
\]

or

\[
\begin{align*}
    \bar{x}_{1.} &= \frac{1}{9}(98 + 81 + 90 + 88 + 83 + 86 + 99 + 96 + 99) = 90 \\
    \bar{x}_{2.} &= \frac{1}{9}(66 + 67 + 67 + 59 + 65 + 76 + 68 + 70 + 74) = 68 \\
    \bar{x}_{2.} &= \frac{1}{9}(107 + 107 + 124 + 110 + 112 + 115 + 124 + 123 + 113) = 115
\end{align*}
\]

Next, we find the calculated sample variances:

\[
\begin{align*}
    s_i^2 &= \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_{i.})^2
\end{align*}
\]

or
The pooled sample variance is an average of the calculated sample variances:

\[ s_p^2 = \frac{1}{N-k} \sum_{i=1}^{k} (n_i - 1)s_i^2 \]

or

\[ s_p^2 = \frac{1}{27 - 3}[(9 - 1)49 + (9 - 1)25 + (9 - 1)49] = 41 \]

Because we have the same number, \( n \), of observations for each sample, we use the simpler form of \( b \) value:

\[ b = \frac{1}{s_p^2} \left( \frac{s_1^2 s_2^2 ... s_k^2}{k} \right) = \frac{(49 \cdot 25 \cdot 49)^{\frac{1}{3}}}{41} = 0.9550 \]

Using \( \alpha = 0.01 \) for a 1 \( \% \) significance-level, the critical value from Table A.10 of [1] is

\[ b_k(\alpha; n) = b_3(0.01, 9) = 0.6676 \]

Since \( b > b_k \), we accept \( H_0 \) and assume that the variances are equal.

**NOTE:** \( b < 1 \) always. For large \( n \), \( b_k(\alpha; n) \) approaches 1.