Ex: Using a spreadsheet or numerical program with observations drawn from a standard gaussian (normal) distribution, (i.e., $\mu = 0$ and $\sigma^2 = 1$), calculate the estimated standard deviation, $\hat{\sigma}$, 15 times based on ranges of 12 samples of 20 observations each. Also, calculate the sample mean and sample standard deviation for these 15 estimated standard deviations.

SOL’N: We use a Matlab® program, (see StatsControlChartXbarEx1.m file), to perform calculations based on the following equations from [1]:

$$R_i = x_{i,\text{max}} - x_{i,\text{min}}$$

$$\bar{R} = \frac{1}{k} \sum_{i=1}^{k} R_i$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

where $k$ = total number of samples being considered

$i$ = index designating sample

$x_{ij}$ = observations available for sample $i$

$d_2$ = number of observations in each sample

$\hat{\sigma}$ = estimated value of $\sigma$

Results for one run of the program are as follows:

$$\hat{\sigma}_1 = 0.969 \quad \hat{\sigma}_2 = 1.032 \quad \hat{\sigma}_3 = 1.057 \quad \hat{\sigma}_4 = 0.974 \quad \hat{\sigma}_5 = 1.072$$

$$\hat{\sigma}_6 = 1.084 \quad \hat{\sigma}_7 = 1.044 \quad \hat{\sigma}_8 = 1.102 \quad \hat{\sigma}_9 = 1.047 \quad \hat{\sigma}_{10} = 0.982$$

$$\hat{\sigma}_{11} = 1.025 \quad \hat{\sigma}_{12} = 1.026 \quad \hat{\sigma}_{13} = 1.028 \quad \hat{\sigma}_{14} = 1.016 \quad \hat{\sigma}_{15} = 0.994$$

The calculated mean of the calculated $\hat{\sigma}$'s is within a few percent of the true $\sigma$, and the standard deviation of the calculated $\hat{\sigma}$'s is only a few percent of the true $\sigma$:

$$\bar{\hat{\sigma}} = 1.0302$$

$$s_{\hat{\sigma}} = 0.0393$$