DESIGN AND PERFORMANCE OF CONSTANT-TEMPERATURE CIRCUITS FOR MICROBRIDGE-SENSOR APPLICATIONS

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ABSTRACT

The performance and transient behavior of constant-temperature microbridge bias circuits is analyzed. We find that this type of circuits must be compensated to prevent excessive overshoot in microbridge temperature. The tradeoff between temperature overshoot and circuit bandwidth is examined.

INTRODUCTION

Electrically-heated microbridges have applications as sensors of flow [1], pressure [2,3], and thermal-conductivity. These devices monitor the heat flow transmitted through the gas surrounding them at a given operating temperature set by a bias circuit. In this paper, we analyze and test the performance of constant-temperature bias and sense circuits applied to microbridges.

Figure 1 shows a sketch of the circuit analyzed. In it a differential amplifier with gain \( G \) and two MOS source followers are connected in a feedback loop. The nonlinear microbridge resistor \( R_b(V_b) \) is connected to the source of M1, and a reference resistor \( R_{ref} \) is connected to the source of M2. The MOSFETS M1 and M2 are ratioed such that their gain ratio \( M_2/M_1 \approx K \).

The operation of the circuit relies on the nonlinear \( I-V \) curve of the microbridge. In the idealized condition (when \( G \to \infty \)), the voltages \( V_1 \) and \( V_b \) are identical; hence the currents through M1 and M2 are ratioed by the factor \( K \) which causes the circuit to adjust \( V_b \) such that the condition \( R_b(V_b) = K \cdot R_{ref} \) is satisfied. The resistance of the microbridge (and its average temperature) is therefore held constant [4]. A desirable property of this circuit is that its output voltage \( V(f) \) is independent of the supply voltages and of fluctuations associated with them.

\[ M_2/M_1 = K \quad \text{(ratioed)} \]

![Figure 1. Constant-temperature microbridge bias circuit](image)

SMALL-SIGNAL ANALYSIS

In this section we find the sensitivity \( A(s) \) of the output voltage \( V_{out} \) for the circuit of Figure 1 subject to a change in the microbridge characteristics induced by changes in its environment.

Figure 2 shows a small-signal equivalent network of the circuit of microbridge small-signal model

\[
\dot{V}_b(s) = g_m G(s) (\dot{V}_b - \dot{V}_{in})
\]

\[
\dot{V}_{in} = \frac{1}{P_h} \frac{\partial V_b}{\partial P_h}
\]

Figure 2. Small-signal equivalent network of the bias circuit of Figure 1.

Figure 1. The microbridge is modeled [5,6] with two resistors, one capacitor, and two voltage generators as shown in the dotted box of Figure 2. The microbridge parameters are

\[ z_b = R_b + \frac{\Delta R_b}{(1 + j \omega \tau_{th})} \quad \text{(1)} \]

\[ \Delta R_b = (R_b - R_{th}) \quad , \quad P_{th} = \frac{1}{\tau_{th}} \quad , \quad C_b = \frac{1}{P_{th} \Delta R_b} \quad \text{(2)} \]

where \( R_b = \frac{\partial V_b}{\partial T} \) is the small-signal microbridge resistance, and \( \tau_{th} \) is its thermal time constant \( \tau_{th} = \rho_m C_m V_m \Theta_T \), where \( \rho_m, C_m \), and \( V_m \) are the microbridge density, heat capacity, and volume. The thermal resistance \( \Theta_T \) is given by the equation

\[ \Theta_T = (\kappa_b \frac{L}{w^2} + \kappa_s \frac{L}{w^2})^{-1} \quad \text{(3)} \]

and represents the thermal heat losses of a bridge of width \( w \), thickness \( z \), length \( l \), gap \( s \), thermal conductivity \( \kappa_b \), and excess flux coefficient \( \eta \) (which accounts for fringing heat flux) immersed in a gas of thermal conductivity \( \kappa_g \) [5,6]. The voltage generators \( \dot{v}_b \) and \( \dot{v}_s \) represent perturbations of the microbridge voltage induced by changes in the gas parameters of interest (flow, pressure, composition) and those induced by a change in the substrate temperature.

In Figure 2, The voltage generator \( \dot{v}_{x+1} \) represents a perturbation of the resistor \( R_{ref} \) voltage induced by a change in the substrate temperature. If both resistor \( R_{ref} \) and microbridge \( R_b \) are made of the same conductive material, they have the same temperature coefficient of resistance.

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If we set the pole \( p_2 > p_b \), the two zeros \( z_{1,2} \) are complex at
\[ s = -\frac{p_b}{2} \left(1 + \frac{\Delta g_b}{R_b}\right) \left(1 \pm j 2\sqrt{n}\right) \]  

and

\[ n = \frac{p_2 \Delta g_b}{p_b R_b - \left(1 + \frac{\Delta g_b}{R_b}\right)^2} \]  

Therefore, the zeros are placed near \( \text{Re}(s) = -p_b/2 \). When the feedback loop is closed, the two poles at \( p_a \) and \( p_b \) move toward the zeros as shown in the root-locus plot of Figure 4. If the op-amp gain \( G(s) \) is sufficiently large, the closed-loop system has a single pole at \( s < -p_b \), and \( p_a \) and \( p_b \) are canceled by the zeros. The remaining pole of \( A(s) \) is real; hence \( \nu_{out} \) has no overshoot. The penalty paid is a decrease in the bandwidth of the opamp output signal since the zero of Eq. (6) is eliminated.

**Figure 4.** Root-locus plot of the open-loop gain \( \Gamma(s) \) of the circuit for \( C_C > 0 \).

### STATE-SPACE ANALYSIS

The results of the small-signal analysis provide an intuitive explanation of the interdependence of the bandwidth of \( A(s) \) and its overshoot with the electrical time constant \( \tau_{el} = \tau_{ref} C_c \) of the circuit and the thermal time constant \( \tau_{th} \) of the microbridge. We now examine this issue in more detail in order to find actual values for the compensation capacitor \( C_C \) that will inhibit the temperature overshoots.

If we assume that \( G \to \infty \), then \( V_b = V_1 \) at all times. In this simplified situation [5], the state variables of the circuit are the average microbridge overtemperature \( \Delta u \) and the capacitor voltage \( V_1 = V_b \). The simplified system of equations that determines the dynamics of the circuit in Figure 1 is

\[ V_1 = -\frac{V_1}{C_c} \left[ \frac{1}{\tau_{ref}} \frac{K}{R_0 (\Delta u)} - V_1 \right] = \frac{V_1}{C_{eq}} \left[ \frac{1}{\tau_{eq}} - \frac{1}{R_0 (\Delta u)} \right] \]  

\[ \rho_m C_m \Delta u = \frac{\Delta u}{\tau_{eq}} + \frac{8}{\tau^2} \left( \frac{V_1}{R_0 (\Delta u)} \right)^2 \]  

where the dot symbolizes a time derivative. These two equations are obtained using the relationship \( R_0 = R_0 (1 + \frac{\Delta u}{\tau_{eq}}) \), and a variational solution of the microbridge heat balance equation [5]. The parameters \( C_{eq} = K^{-1} C_c \) and \( \tau_{eq} = K \tau_{ref} \) are defined to simplify them.

The system of Eq. (13) has equilibrium points at the origin and at

\[ \Delta u^* = \frac{1}{4} \left( \frac{R_{ref}}{R_0} - 1 \right), \quad V_1^* = \frac{1}{8} \left( \frac{R_{eq} - R_0}{\tau_{eq} R_0} \right)^2 \]  

The dynamics of the system are best understood if the system is first non-dimensionalized. Letting

\[ x_1 = \frac{V_1}{V_{1*}}, \quad x_2 = \frac{\Delta u}{\Delta u^*}, \quad \tau = \frac{t}{\tau_{th}} \]  

and substituting into Eq. (13), we obtain the reduced system

\[ \frac{dx_1}{dt} = -\frac{\alpha \beta x_1 (x_2 - 1)}{(1 + \beta x_2)} = f_1(x_1, x_2) \]  

\[ \frac{dx_2}{dt} = -x_2 (1 + \beta x_2) = f_2(x_1, x_2) \]  

with

\[ A(s) = \frac{A(0)}{(1 + \frac{s}{p_a}) (1 + \frac{s}{p_b})} \]  

\[ A(0) = \frac{g_{in} G(0) R_b}{(g_{in} R_b - R_b) G(0) + 1} \]  

or

\[ \nu_{out}(s) = A(s) \nu_{in}(s) - \nu_i(s) \]  

where \( A(s) \) is

\[ A(s) = \frac{A(0)(1 + \frac{s}{p_a}) (1 + \frac{s}{p_b})}{(1 + \frac{s}{p_a}) (1 + \frac{s}{p_b})} \]  

\[ s = \frac{(p_a + p_b)}{2} \left[ 1 \pm \left(1 - \frac{4 \Delta g_b G(0) p_a}{R_2 (p_a + p_b)} \right)^{1/2} \right] \]  

Figure 3. Pole-zero plot for the closed loop gain \( A(s) \) as given in Eq. (6).

\[ \text{TSC} \, \Delta u \, \text{and hence} \, v_{out} \text{has no overshoot. The penalty paid is a decrease in the bandwidth of the op-amp output signal since the zero of Eq. (6) is eliminated.} \]
or in vector form \( \dot{x} = f(x) \). The parameters \( \alpha \) and \( \beta \) of Eq. (16) are

\[
\alpha = \frac{\rho C_m V_m \Theta_r}{R_{eq} C_{eq}} - \frac{\tau_{th}}{\tau_{el}}, \quad \beta = \left( \frac{R_{eq}}{R_1} - 1 \right)
\]

(17)

The parameter \( \alpha \) is the ratio of the microbridge thermal time constant \( \tau_{th} \) to the circuit electrical time constant \( \tau_{el} \), and \( \beta \) represents the equilibrium microbridge overtemperature.

The system of Eq. (17) has equilibrium points at the origin and at \((1,1)\). Linearizing the non-dimensional system at \((1,1)\), we obtain

\[
\dot{x} = J \hat{x}
\]

(18)

where \( J \) is the Jacobian of \( f(x) \) evaluated at \((1,1)\). The behavior of the linearized system is determined by the eigenvalues of \( J \) which are

\[
\lambda_{1,2} = -\frac{1}{2} \left( 1 + \frac{2\beta}{1 + \beta} \right) \pm \frac{\sqrt{\left( 1 + \frac{2\beta}{1 + \beta} \right)^2 - \frac{4\alpha\beta}{(1 + \beta)^2}}}{2}
\]

(19)

Note that both eigenvalues have negative real parts. The overshoot of the circuit can be eliminated if both eigenvalues are real. This requires that the term inside the square root term of Eq. (19) be greater than zero; hence

\[
\frac{\tau_{th}}{\tau_{el}} = \alpha < \frac{(1 + 2\beta)^2}{8(1 + \beta)^2}
\]

(20)

or equivalently

\[
C_c > \frac{\rho C_m V_m \Theta_r}{R_{ref}} \frac{8\beta(1 + \beta)}{(1 + 2\beta)^2} = \frac{8\beta \rho \nu C_c V_m \Theta_r}{R_1}, \quad \beta \ll 1
\]

(21)

Figures 5(a) and (b) show the state-space plots of the system of Eq. (16).

Figure 6. State-space trajectories for underdamped and overdamped system.

Figure 7 shows experimental values of the gain \( A(j\omega) = \hat{v}_o / \hat{v}_i \) as a function of frequency for several values of \( C_c \) for a bias circuit with a 400 \times 3 \times 1 \text{µm}^3 \) microbridge operating at about 100 °C. The gas-induced signal \( \hat{v}_g \) was simulated with the test voltage generator \( \nu_{test} \). The measurements were performed with a HP4195 network analyzer. If the capacitor \( C_c = 0 \), the measured gain has one zero and two complex poles as predicted by Eq. (6). The zero is responsible for the initial increase in the gain, which is sometimes desirable because it helps to cancel the intrinsic thermal-heating pole \( \nu_{th} \) of \( \hat{v}_g \) \cite{8}. A low value of \( C_c \), however, can lead to large oscillations and overheating of the microbridge. Such overheating can result in an irreversible degradation \cite{7} and a change of \( V_{out} \) every time the circuit is powered up. Note that as \( C_c \) is increased, the transfer function \( A(j\omega) \) loses its zero and becomes a single pole function as predicted by the root locus analysis. Figures 8(a)-(b) show photographs of the microbridge voltage \( V_1(t) \) during power-up for

\[\text{MEASUREMENTS}\]

\[\text{Figure 6. State-space trajectories for underdamped and overdamped system.}\]

\[\text{Figure 7. Experimental values of the circuit gain } A(j\omega).\]

\[\text{Figure 8(a)-(b) show photographs of the microbridge voltage } V_1(t) \text{ during power-up for}\]
two different values of $C_e$. In Figure 8(a), the circuit is underdamped (low $C_e$) and during the transient the microbridge voltage reaches up to the supply limit. In Figure 8(b), corresponding to high $C_e$, there is no observable voltage (and temperature) overshoot.

![Figure 8](image)

**Figure 8.** Measured microbridge voltage during power-up: (a) underdamped system, (b) overdamped system.

CONCLUSIONS

The performance and transient behavior of constant-temperature microbridge bias circuits are analyzed and measured. This type of circuit enhances the microbridge gas-induced signals with respect to those induced by the substrate-temperature. We find that this type of circuits must be compensated to prevent excessive microbridge overheating. The elimination of temperature overshoot implies a reduction in the circuit bandwidth.

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References