Final Exam.
With Solutions

Name ______________________
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UNIVERSITY OF UTAH

ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

MICROWAVE ENGINEERING I

ECE 5320/6322

FINAL EXAMINATION

December 15, 2014

1. (30 points)

Calculate the unloaded Q of a short-circuited microstripline resonator of length $\ell = \lambda_g / 4$ and characteristic impedance $Z_o' = 30$ ohms at 5000 MHz. For this circuit:

Pts		
10	a.	Calculate the physical length ℓ assuming a substrate of dielectric constant $\varepsilon_r = 2.5$ and thickness 1/16".
15	b.	Calculate the unloaded Q at the resonant frequency of 5000 MHz assuming copper as the material for the microstrip and the loss tangent for the dielectric substrtate $\tan \delta = 0.001$.
5	c.	Without calculating individual R, L, and C values, draw the equivalent circuit of this resonator.

1.

b. The equations for RLC eqvt cct. of this short-circuited $l = \lambda_g/4$ resonator are given as Eqs. 6.30a, 6.30b and 6.30c, respectively

$$R = \frac{Z_0}{\alpha l} \ (6.30a) \ ; \ C = \frac{\pi}{4\omega_0 Z_0} \ (6.30b) \ \text{and}$$

$$L = \frac{1}{\omega_0^2 C} \ (6.30c)$$

Furthermore $Q_0 = \dfrac{\beta}{2\alpha}$ (6.31)

From Eqs. 3.195 – 3.199 of the Text, we need to calculate the parameters for a $Z_0 = 30$ ohm microstrip line printed on a $\epsilon_r = 2.5$ substrate of thickness $d = 1/16''$

For this line

$$A = \overbrace{\frac{Z_0}{60}\sqrt{\frac{3.5}{2}}}^{0.6614} + \frac{1.5}{3.5}\overbrace{\left(0.23 + \frac{0.11}{2.5}\right)}^{0.274} = 0.779$$

Since A is less than 1.52, the second part of Eq. 3.197 should be used. Hence we should also calculate B

$$B = \frac{377\pi}{2 Z_0\sqrt{\epsilon_r}} = \frac{377\pi}{2\times 30\sqrt{2.5}} = 12.484$$

$$\frac{W}{d} = \frac{2}{\pi}\left[11.484 - \underbrace{\ln(23.968)}_{3.177} + \underbrace{\frac{1.5}{5}\left\{\underbrace{\ln(11.484)}_{2.441} + \overbrace{0.39 - \frac{0.61}{2.5}}^{0.146}\right\}}_{0.6885}\right]$$

$$= \boxed{5.727}$$

From Eq. 3.195

$$\epsilon_e = \frac{3.5}{2} + \frac{1.5}{2}\underbrace{\frac{1}{\sqrt{1+\frac{12}{5.727}}}}_{0.4263} = \boxed{2.176}$$

$$\boxed{\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_e}} = \frac{6}{\sqrt{2.176}} = 4.067 \text{ cm} = 4.067\times 10^{-2}\text{ m}}$$

$$\beta = \frac{2\pi}{\lambda_g} = \frac{2\pi\sqrt{\epsilon_e}}{\lambda_0} = \frac{2\pi\sqrt{\epsilon_e}}{6} = \frac{2\pi}{4.067\times 10^{-2}}$$

$$= 154.49 \text{ m}^{-1}$$

a. $\boxed{l = \lambda_g/4 = 1.016 \text{ cm}}$

Converting 1.6 dB/m into nepers/m, we can write

$$\alpha = \frac{1.6}{8.686} = 0.1842 \text{ nepers/m}$$

From Eq. 6.31

$$Q_0 = \frac{\beta}{2\alpha} = \frac{154.49}{2\times 0.1842} = \boxed{419.34}$$

$\boxed{\text{c.}}$ EQVT cct.

2. (30 points)

The S-matrix of a three-port circuit measured with a Zo = 50 ohm measurement system is given as follows:

$$S = \begin{bmatrix} 0.2 & 0.2 & 0.5 \\ 0.6 & 0.3 & 0.5 \\ 0.4 & 0.5 & -0.4 \end{bmatrix}$$

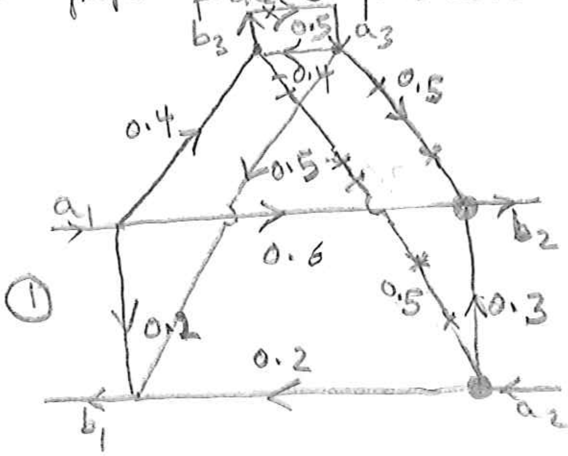
<u>Pts</u>

10 a. Draw the flow graph for the above circuit.

15 b. Calculate the reflection coefficient at port 2 if port 3 is connected to a mismatched load with a reflection coefficient of 0.5 and port 1 is terminated (connected to a matched load).

5 c. Calculate the input impedance (magnitude and phase) of the circuit at port 2 when ports 1 and ~~2~~ 3 are connected as given in part b.
Hint: Do not draw any branches with value zero.

2. a. The flow graph of the 3-port circuit is sketched below



b. Paths between a_2 and b_2

Note that the only loop in this circuit is at port ③

P	Value	First order non-touching loops $L^{i}(1)$	Second order non touching loops $L^{i}(2)$
P_1	0.3	$(0.5)(-0.4) = -0.20$	0
P_2	$0.5\times0.5\times0.5$	0	

$$\Gamma_2 = \frac{b_2}{a_2} = \frac{\overset{P_1}{0.3\left[1+0.2\right]} + \overset{P_2}{0.5\times0.5\times0.5\left[1-0\right]}}{\left[1+0.2\right]}$$

$$= 0.3 + \frac{0.125}{1.2} = 0.4042$$

c. $$\Gamma_2 = \frac{Z_2 - 50}{Z_2 + 50} = 0.4042$$

$$\frac{1+|\Gamma_2|}{1-|\Gamma_2|} \rightarrow Z_2 = Z_0\,\frac{1.4042}{1-0.4042} = 50\times2.3576 = 117.84\ \Omega$$

Ohms

Zero phase

3. (25 points)

Pts	
10	a. Design a full π-section m-derived low-pass filter with the following specifications: $R_o = 60\ \Omega$, $f_c = 5.0$ GHz, $f_\infty = 6.0$ GHz.
8	b. Calculate the values of series inductance and shunt elements to use for this full π-section m-derived low-pass filter.
7	c. Calculate the image impedance Z_π of the filter at 3.0 and 4.0 GHz, respectively.

4. a. From Chapter 3 Notes (also Eq. 3.86 of the Text)

$$Z_{TE} = \frac{\omega\mu}{\beta} = \frac{\eta\ 377}{\sqrt{1-\left(\frac{f_c^2}{f^2}\right)}} = 480$$

$$1-\left(\frac{f_c}{f}\right)^2 = \left(\frac{377}{480}\right)^2 = 0.6169$$

$$\frac{f_c}{f} = \sqrt{1-0.6169} = 0.619$$

$$f_c = (15 \times 0.619) = 9.284 \text{ GHz}$$

b. $f_c\big|_{TE_{10}} = \frac{c}{2a} = 9.284 \times 10^9$

$$a = \frac{30}{2 \times 9.284} = 1.6156 \text{ cm} \Rightarrow 0.636''$$

c. $$f_c\Big|_{TE_{11}} = \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = \frac{c}{2a}\sqrt{7.25} = \frac{15\times 10^9}{2 \times 1.6156}\sqrt{7.25}$$

$$= \boxed{25 \text{ GHz}}$$

$$f_c\Big|_{TE_{12}} = \frac{c}{2a}\sqrt{1+(2\times 2.5)^2} = f_c\Big|_{TE_{10}} \times \sqrt{26} = \boxed{47.34 \text{ GHz}}$$

d. $$\Gamma = \frac{Z_L - Z_{TE}}{Z_L + Z_{TE}} = \frac{400-480}{880} = -\frac{80}{880} = 0.091\angle 180^\circ$$

e. For the dielectric filled waveguide

$$f_c' = \frac{c_\epsilon}{2a} = \frac{9.284}{\sqrt{2.5}} = \boxed{5.872 \text{ GHz}}$$

$$Z_{TE}\Big|_{\text{dielectric filled WG}} = \frac{\eta_\epsilon}{\sqrt{1-\left(\frac{f_c'}{f}\right)^2}} = \frac{377/\sqrt{2.5}\ \ 238.4375}{\sqrt{1-\left(\frac{5.872}{15}\right)^2}\ \ \sqrt{0.84675}} = \boxed{259.1 \text{ Ohms}}$$

4. (30 points)

The wave impedance of the TE_{10} mode of an air-filled rectangular waveguide at a frequency of 15 GHz is given to be 480 ohms.

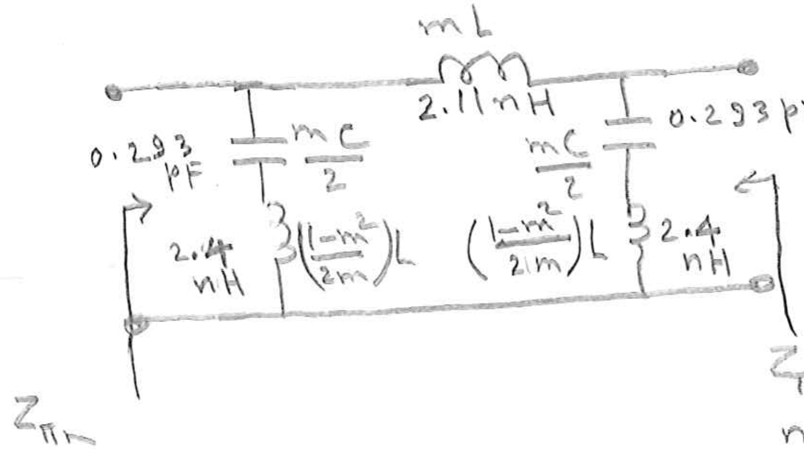
Pts	
5	a. Calculate the cutoff frequency for the TE_{10} mode of this waveguide.
4	b. Calculate the "a" dimension for the waveguide.
8	c. Calculate the cutoff frequencies for the TE_{11} and TE_{12} modes for this waveguide given that **b = 0.4 a.**
5	d. **Without using the Smith chart**, calculate the reflection coefficient Γ (**magnitude and phase**) in the air-filled waveguide if it is connected to a load of effective impedance Z_L = 400 ohms.
8	e. Calculate the wave impedance at the signal frequency of 15 GHz for the TE_{10} mode if the above waveguide is filled with a dielectric with $\varepsilon_r = 2.5$. Note that the cutoff frequency for the TE_{10} mode of a dielectric-filled waveguide is lower than that for an air-filled waveguide.

3. From Eq. 8.44 of the Text

$$f_\infty = \frac{f_c}{\sqrt{1-m^2}}$$

For this filter $\quad 1-m^2 = \left(\frac{f_c}{f_\infty}\right)^2 \Rightarrow m = \sqrt{1-\left(\frac{5}{6}\right)^2} = \frac{\sqrt{11}}{6} = \boxed{0.553}$

The sketch of a full π-section m derived filter can be written from Table 8.2 of the Text



From Ex. 8.2 of the Text

$$L = \frac{2R_o}{\omega_c} = \frac{60}{\pi \times 5 \times 10^9} = \boxed{3.82 \text{ nH}}$$

$$C = \frac{2}{R_o \omega_c} = \frac{2 \cancel{10^{12}}}{60 \times 2\pi \times 5 \times \cancel{10^9}} = \frac{1000}{300\pi} = \boxed{1.061 \text{ pF}}$$

$$\frac{mC}{2} = 0.293 \text{ pF}$$

$$\frac{(1-m^2)}{2m} L = \frac{0.694}{2 \times 0.553} \times 3.82 = 2.398 \sim 2.4 \text{ nH}$$

From Eq. (8.46) of the Text

$$Z_{i\pi} = R_o \frac{1-(f/f_\infty)^2}{\sqrt{1-(f^2/f_c^2)}}$$

@ 3 GHz $\quad Z_{i\pi} = 60 \dfrac{1-\left(\frac{3}{6}\right)^2}{\sqrt{1-(3/5)^2}} = 60 \dfrac{0.75}{0.8} = 56.25$ Ohms

(0.36)

@ 4 GHz $\quad Z_{i\pi} = 60 \dfrac{1-\left(\frac{4}{6}\right)^2}{\sqrt{1-(0.8)^2}} = 60 \dfrac{\frac{20}{36}}{0.6} = 55.56$ Ohms

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Score:

Problem 1	________	of a possible 30 points
Problem 2	________	of a possible 30 points
Problem 3	________	of a possible 25 points
Problem 4	________	of a possible 30 points
Total	________	of a possible 115 points