Additional Pages of Notes for Chapter 7

T-Junction Unequal Power Dividers

See also Ex. 7.1 of the Text

Question:

Can we show that the reciprocity of S-parameters ( $S_{ij} = S_{ji}$ ) holds even though the circuit of Fig. 7.6 for a lossless, but not symmetric, power divider T-junction is not symmetric with respect to ports 1, 2, 3.

Proof: The T-junction of Fig. 7.6 of the Text p. 316 is shown below.

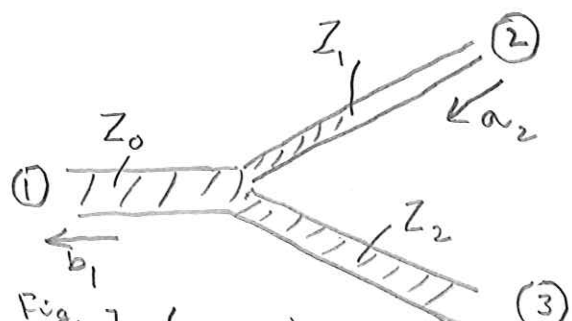


Fig. 7-6 on p.316 of the Text

For this circuit let us mark ports 1, 2, 3 as shown here. For Example 7.1 we can match only port 1 with $Z_0 = 50\,\Omega$ provided we use $Z_1 = 3Z_0 = 150\,\Omega$ and $Z_2 = \frac{3Z_0}{2} = 75\,\Omega$ and this circuit results in a lossless 2:1 ratio power divider with power input $P_{in}$ at port 1 resulting in $\frac{1}{3}P_{in}$ power output from port 2 and $\frac{2}{3}P_{in}$ power output from port 3. We can write the S-matrix for this circuit as follows:

$$[S] = \begin{bmatrix} 0 & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & -2/3 & \sqrt{2}/3 \\ \sqrt{\frac{2}{3}} & \sqrt{2}/3 & -1/3 \end{bmatrix} \quad (1)$$

In writing Eq. (1) I have used the power divisions $|S_{21}|^2 = 1/3$; $|S_{31}|^2 = 2/3$, $\Gamma_2 = S_{22} = -2/3$; $\Gamma_3 = S_{33} = -1/3$ derived on p. 317 of the Text.

I have also used $S_{21} = S_{12} = \frac{1}{\sqrt{3}}$, $S_{31} = S_{13} = \sqrt{2/3}$ and the lossless circuit properties 1. Sum of the squares of S-parameters in any column = 1

2. Sum of the S-parameters in any column multiplied by the conjugate of the

Corresponding S-parameters in any other column = 0

In the following I would show as an example that $S_{21} = S_{12} = \frac{1}{\sqrt{3}}$ even though ports ① and ② use different characteristic impedances $Z_0 = 50\,\Omega$, $Z_1 = 150\,\Omega$ and are not symmetrical.

Note that from Eq.(1) we can write

(2) $$b_1 = \frac{1}{\sqrt{3}} a_2$$ when power is input to port ② and port ③ is terminated i.e. $a_3 = 0$

From Eq. (2) we can write $S_{12} = \left.\frac{b_1}{a_2}\right|_{a_3=0} = \frac{1}{\sqrt{3}}$

Is this true?

For signal $a_2$ into port ②, the load is presented by a parallel Combination of $Z_0$ and $Z_2$ i.e. $50\,\Omega \parallel 75\,\Omega$ which is a load impedance of $37.5\,\Omega$. This gives a reflection coefficient $S_{22} = -2/3$ as derived in the Text.

Power delivered to the effective load of $37.5\,\Omega$ i.e. $50\,\Omega \parallel 75\,\Omega$ is $a_2 a_2^* - b_2 b_2^* = a_2 a_2^* \left(1 - |S_{22}|^2\right) = \frac{5}{9} a_2 a_2^*$

This power delivered to the junction is split two ways

out of port ① $$b_1 b_1^* = \left(\frac{75}{50+75}\right) \times \left(\frac{5}{9} a_2 a_2^*\right) = \frac{a_2 a_2^*}{3}$$

This gives $|S_{12}| = \left|\frac{b_1}{a_2}\right| = \frac{1}{\sqrt{3}}$

out of port ③ $$b_3 b_3^* = \left(\frac{50}{50+75}\right) \times \left(\frac{5}{9} a_2 a_2^*\right) = \frac{2}{9} a_2 a_2^*$$

This gives $|S_{32}| = \left|\frac{b_3}{a_2}\right| = \frac{\sqrt{2}}{3}$

Both of these terms were used in writing Eq.(1) for the S-matrix.